203. *On Potent Rings. I*[*)]

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A ring R is said to be (right) *potent* iff every nonzero closed right ideal A of R is potent, that is, A^n is not zero for all positive integer n. In [6], R. E. Johnson has investigated potent irreducible rings which are finite dimensional in the sense of Goldie [4], and obtained many interesting results. The aim of this paper is to generalize the Johnson's work [6] to the case of the rings with infinite dimensions.

1. Definitions and notations.

Let R be an associative ring. A right ideal I of R is called *closed* if it has no proper essential extensions in R as right R-modules. Clearly the concept of closed right ideals of R coincides with the one of complemented right ideals in the sense of Goldie [4]. A right ideal E of R is called *large* if R is an essential extension of E (in symbols; $E\subset' R$). A ring R is said to be *(right) locally uniform* if any nonzero right ideal of R contain a nonzero uniform right ideal. A right ideal A is *uniform* if A is an essential extension of every nonzero right ideal contained in A. Clearly, if R is finite dimensional, then R is locally uniform. R is called *countably dimensional* if R has a direct sum of countable right ideals. The notation $A^r(A^l)$ is used for right (left) annihilator of a subset A of R.

The set $Z_r(R)=\{x\in R\mid x^r:\text{ large right ideal of } R\}$ is an ideal of the ring R, which is called the *right singular ideal.* If $Z_r(R)=0$, then the each right ideal A has a unique maximal essential extension A^* in R. The set $L_r^*(R)(=L_r^*)$ of closed right ideals is a complete complemented modular lattice under the inclusion. If $\{C_i\mid i\in I\}$ is any collection of closed right ideals of R, then $\bigcup_{i\in I}^* C_i=(\sum_{i\in I} C_i)^*$. $(J_r^*;\cap,\cup)$ will denote the lattice of all annihilator right ideals of R. Then it is easily seen that $J_r^*\subseteq L_r^*$. We note that the lattice J_r^* is not usually a sublattice of L_r^*, although intersections are set-theoretic in both lattices. For convenience, we let $L_{r2}^*=L_r^*\cap L_2$ and $J_{r2}^*=J_r^*\cap L_2$, where L_2 is the set of two-sided ideals of R. Corresponding left properties of a ring R are indicated by replacing each "r" by an "l". If R is right locally uniform, then L_r^* is an atomic lattice, and $A\in L_r^*$ is an atom if and only if A is a closed uniform right ideal. Following R. E. Johnson we call

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