226. *On Realization of the Discrete Series for Semisimple Lie Groups*

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This note is an announcement of a result, which says, briefly, that most of the discrete series for a semisimple Lie group are realized as certain eigenspaces of the Casimir operator on the symmetric space (Theorem 2). This construction is in some sense a generalization of the methods adopted in [1], [2], [9] for special groups and in [5] for the groups of hermitian type. Also, [6] indicates the above method of realization. Further, as for alternative methods to realize most of the discrete series, we refer to the recent works [5], [8]. Our technique used here depends heavily on that of [5]. A detailed exposition with full proofs will appear elsewhere.

1. Let G be a connected non-compact semisimple Lie group with a compact Cartan subgroup. We assume, for convenience, that G has a faithful finite dimensional representation and its complexification G^C is simply connected. Fix a maximal compact subgroup K of G and a Cartan subgroup H contained in K. We denote by $\mathfrak{g}, \mathfrak{k}$ and $\mathfrak{h}$ the Lie algebras corresponding to G, K and H respectively. For complexifications $\mathfrak{g}^C, \mathfrak{k}^C, \mathfrak{h}^C$ of $\mathfrak{g}, \mathfrak{k}, \mathfrak{h}$, we denote by Δ the root system of $(\mathfrak{g}^C, \mathfrak{h}^C)$, and by W_G the Weyl group of $(\mathfrak{k}^C, \mathfrak{h}^C)$. Taking a positive root system P of Δ fixed once for all, P_k (resp. P_n) denotes the set of a positive compact (resp. non-compact) roots. Let L be the character group of H, L' the set of regular elements in L. Introducing an inner product (,) on L induced by the Killing form, we put $\varepsilon(\lambda)=\text{sign}\prod_{\alpha\in P}(\lambda,\alpha)$ for $\lambda\in L'$, and $\varepsilon(\lambda)=0$ for $\lambda\in L-L'$. We also put $\varepsilon_k(\lambda)=\text{sign}\prod_{\alpha\in P_k}(\lambda,\alpha)$ if $\lambda\in L$ is $\mathfrak{k}^C$-regular, and $\varepsilon_k(\lambda)=0$ if λ is $\mathfrak{k}^C$-singular. For discrete series, the following fact is known by Harish-Chandra [3]. Let $\mathcal{E}_d$ be the discrete series for G. For $\lambda\in L'$, there then exists a unique element $\omega(\lambda)\in\mathcal{E}_d$, and the map $L'\ni\lambda\mapsto\omega(\lambda)\in\mathcal{E}_d$ is surjective, while $\omega(\lambda)=\omega(\lambda')$ if and only if there exists $w\in W_G$ such that $w\lambda=\lambda'$. We shall denote by $\Theta_{\omega(\lambda)}$ the character of $\omega(\lambda)$.

For a finite subset A of L, we shall denote by $|A|$ its cardinal number and put $\langle A\rangle=\sum_{\alpha\in A}\alpha$. Put $\rho=\langle P\rangle/2, \rho_k=\langle P_k\rangle/2$ and $\rho_n=\rho-\rho_k$. If $\varepsilon_k(\lambda+\rho_k)\neq 0$ for $\lambda\in L$, there exists a unique $w\in W_G$ such that $w(\lambda+\rho_k)-\rho_k$ is k^C-dominant. We then denote by $[\lambda]$ the equivalence class to which belongs an irreducible K-module with highest weight