256. Pseudo-differential Operators in the Theory of Hyperfunctions

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In this note we first give the definition of the sheaf of pseudo-differential operators, the notion of which is originally due to Sato. The sheaf is defined using Sato's theory of the sheaf $\mathcal{C}$. For the definition and the main properties of the sheaf $\mathcal{C}$ we refer the reader to Sato [2].

Secondly we develop the theory of pseudo-differential operators of finite type and discuss some applications to the theory of linear partial differential operators.

This note is a summary of our forthcoming paper in which the details will be given.

1. Let f be a real analytic mapping from an n-dimensional real analytic manifold N to an m-dimensional real analytic manifold M. The mapping f defines a natural homomorphism of vector bundles: $N\underset{M}{\times} T^*M \rightarrow T^*N$, where T^*M and T^*N denote the cotangent bundle of M and that of N, respectively, and $N\underset{M}{\times} T^*M$ denotes the fibre product of T^*M and N over M. We denote its kernel by T_N^*M, which is not a vector bundle in general, and define S_N^*M to be $(T_N^*M-N)/\boldsymbol{R}^+$. ($\boldsymbol{R}^+$ denotes the multiplicative group of positive numbers.) S_N^*M is a closed subset of $N\underset{M}{\times} S^*M$, whose fibres over N are spheres. The $f^*: N\underset{M}{\times} T^*M \rightarrow T^*N$ induces a projection $\rho: N\underset{M}{\times} S^*M - S_N^*M \rightarrow S^*N$. We denote by $\tilde{\varpi}$ the natural projection $N\underset{M}{\times} S^*M - S_N^*M - S^*M$.

Let $\mathcal{A}_M$ be the sheaf of real analytic functions and let v_M be the sheaf of densities with real analytic coefficients, which becomes an invertible $\mathcal{A}_M$-Module. By means of the theory of $\mathcal{C}$, we can define the following two fundamental homomorphisms, corresponding to the substitution and integration along fibre.

$$f^*: \rho_! \tilde{\varpi}^{-1}\mathcal{C}_M \rightarrow \mathcal{C}_N,$$
$$f_*: \tilde{\varpi}_! \rho^{-1}(\mathcal{C}_N \underset{\mathcal{A}_N}{\otimes} v_N) \rightarrow \mathcal{C}_M \underset{\mathcal{A}_M}{\otimes} v_M.$$

Remark. If $f: X \rightarrow Y$ is a continuous map and $\mathcal{F}$ is a sheaf on X, $f_!(\mathcal{F})$ is a direct image with proper support, that is, $\Gamma(U, f_!(\mathcal{F}))$

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