255. *The Decomposition of $L^2(\Gamma \backslash SL(2,R))$ and Teichmüller Spaces*

By Noriaki KAWANAKA
Department of Mathematics, Osaka University

(Comm. by Kunihiko KODAIRA, M. J. A., Dec. 12, 1970)

0. Let Γ be a discrete subgroup of $G=SL(2,\boldsymbol{R})/\{\pm e\}$ and let χ be a finite dimensional representation of Γ by unitary matrices. We assume that $\Gamma\backslash G$ is compact.

It is well known that the unitary representation $U^{\Gamma,\chi}$ of G induced from χ can be decomposed into the discrete direct sum $\sum_i \oplus U_i$ of irreducible unitary representations U_i of G. We call the set $\{U_i\}$ the spectra of $U^{\Gamma,\chi}$.

The problem we want to study is the following[1]:

"*How do the spectra of $U^{\Gamma,\chi}$ behave when Γ varies?*"

Detailed proofs will appear elsewhere.

1. Let $H=\{z=x+iy\,;\,y>0\}$ be the complex upper half plane. G acts on H transitively by

$$g(z)=\frac{az+b}{cz+d}$$

for z in H and $g=\begin{pmatrix} a & b \\ c & d\end{pmatrix}$ in G.

The G-invariant metric on H is

$$ds^2=\frac{dx^2+dy^2}{y^2}.$$

Hence, the G-invariant measure on H is

$$dm(z)=\frac{dxdy}{y^2}$$

and the ring of G-invariant differential operators on H is generated by

$$\Delta=-y^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right). \tag{1}$$

2. Let V be the representation space of χ. Consider the complex vector space $\mathcal{H}(\Gamma,\chi)$ of all V-valued functions F on H wich satisfy the following conditions:

(i) F is (componentwisely) measurable;

(ii) $F(AZ)=\chi(A)F(z)$ for all z in H and A in Γ;

(iii) $\int_{\mathcal{F}} {}^tF(z)\overline{F(z)}dm(z)<\infty$ where $\mathcal{F}$ is a measurable fundamental

1) Note that some problems of the similar nature were also discussed by J. M. G. Fell [3].