253. *Stable Properties of Gaussian Flows*

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1. It is important to study the stability of dynamical systems as a generalization of mixing property. The strong and the weak stabilities for an automorphism on a probability space were studied by A. Maitra [3] and by S. Natarajan and K. Viswanath [4] (cf. Renyi [5]).

In this paper we shall study the stabilities (mixing property) of a Gaussian flow (flow of the Brownian motion) together with skew product flow of it and a measurable flow with pure point spectrum. As will be seen later, the stabilities coincide with the corresponding mixing properties on each ergodic part of a given dynamical system. Anzai's method in [1] and [2] of skew product dynamical systems is very useful to construct some kinds of models in ergodic theory. In fact we shall be able to give some characteristic properties of a Gaussian process and a Brownian motion by using such a method in § 3 and § 4.

2. Let $(\Omega, \mathcal{B}, m)$ be a probability measure space on which a measurable flow $\{T_t\}$ is given and $\{U_t\}$ denote the one parameter group of unitary operators induced by $\{T_t\}$.

Definition 1. A flow $\{T_t\}$ is said to be *weakly stable* if there exists a constant $C(f, g)$ such that

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T |(U_t f, g) - C(f, g)|\, dt = 0 \tag{1}$$

holds for arbitrary bounded measurable functions f and g.

Definition 2. A flow $\{T_t\}$ is called *strongly stable* if there exists a constant $C(f, g)$ such that

$$\lim_{T\to\infty} (U_t f, g) = C(f, g) \tag{2}$$

holds for arbitrary bounded measurable functions f and g.

Definition 3. Let $(f_0, f_1, \cdots, f_r)$ be an arbitrary $(r+1)$-tuple of bounded functions of $L^2(\Omega)$ and $(t_0^n, t_1^n, \cdots, t_r^n)$ be an arbitrary $(r+1)$-tuple of real numbers satisfying the condition:

$$t_0^n < \cdots < t_r^n \text{ and } \lim_{n\to\infty} \min_{1\leq j\leq r} (t_j^n - t_{j-1}^n) = \infty. \tag{3}$$

We call $\{T_t\}$ an *r-order stable* flow if there exists a constant $C(f_0, \cdots, f_r)$ such that

$$\lim \left(\prod_{j=0}^{r} U_{t_j^n} f_j, 1 \right) = C(f_0, \cdots, f_r). \tag{4}$$