249.   Some Characterizations of Regular Duo Rings and Semigroups

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Some ideal-theoretic characterizations of regular duo semigroups[1] and of strongly regular rings (=regular duo rings) were given by the author [2]-[5], and by the author and F. Szász [6]-[8]. In this note we shall establish several further ideal-theoretic characterizations of these classes of associative rings and semigroups.

First we prove the following criterion.

Theorem 1. *A semigroup S is a regular duo semigroup if and only if the relation*

$$(1) \qquad\qquad L \cap R = LRS$$

holds for every left ideal L and every right ideal R of S.

Proof. Let S be a semigroup with property (1) for any left ideal L and any right ideal R of S. Then (1) implies

$$(2) \qquad\qquad R = SRS$$

for any right ideal R of S, i.e. every right ideal R of S is two-sided. Similarly, (1) implies

$$(3) \qquad\qquad L = LS^2$$

for each left ideal L of S, that is each left ideal L of S is two-sided. Therefore S is a duo semigroup. Next we show that S is regular. For any (two-sided) ideal I of S (1) implies

$$(4) \qquad\qquad I = I^2 S = IS^2 = SIS.$$

Hence we get

$$(5) \qquad\qquad I^2 = (SIS)(SIS) = SI,$$

and

$$(6) \qquad\qquad I^2 = I(IS^2) = (I^2 S)S = IS$$

for every ideal I of S. (5) and (6) imply

$$(7) \qquad\qquad IS = SI$$

for any ideal I of S. Finally (4) and (7) imply the relation

$$(8) \qquad\qquad I = ISI$$

for each ideal I of S. This guarantees the regularity of S (cf. Luh [9]).

Conversely, let S be a regular duo semigroup. Then we have

$$(9) \qquad\qquad I_1 \cap I_2 = I_1 I_2$$

for any couple of (two-sided) ideals of S. (9) implies

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1)   We adopt the notation and terminology of [1].