247. On an Algebraic Model for von Neumann Algebras. II

By Hiroshi TAKAI
Department of Mathematics, Osaka Kyoiku University

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1. In the previous paper [6], we have introduced algebraic models of von Neumann algebras as a non-commutative extension of algebraic models of measures due to Dinculeanu and Foiaş [2], and we have proved that an algebraic model determines the algebra structure up to isomorphisms (unfortunately, a von Neumann algebra can not decide an algebraic model within isomorphisms).

In an another note [3], Dinculeanu and Foiaş, introduced an another concept, that is, algebraic models for measure preserving transformations. In § 2, we shall extend the above notion to an automorphism of von Neumann algebra. In § 3, we shall present a measure system which is an algebraic model for the crossed product of a von Neumann algebra by an automorphism group. And in § 4, we shall present an algebraic ergodic system which is an algebraic model for a certain automorphism of the crossed product.

Dinculeanu and Foiaş [3] introduced also the notion of discrete models of measures and established several important theorems. A non-commutative variant of discrete models gives a characterization of group von Neumann algebras. We shall discuss discrete models in a subsequent paper.

Throughout the note, we shall use the terminology of [4] without explanations.

2. Let (Γ, φ) be a measure system introduced in [6], U be an isomorphism of Γ into Γ. Then we shall say (Γ, U, φ) an *algebraic ergodic system* if

$$\varphi(U\gamma)=\varphi(\gamma) \qquad \text{for every} \quad \gamma \in \Gamma.$$

Two algebraic ergodic systems (Γ, U, φ) and (Δ, V, φ) are called *isomorphic* if there exists an isomorphism ϕ of Γ into Δ such that

(i) $$\varphi(\gamma)=\psi(\phi\gamma)$$

and

(ii) $$\phi U=V\phi.$$

Definition 1. Let (Γ, φ) be an algebraic model for a von Neumann algebra $\mathcal{A}$ with a generating vector x and U an isomorphism of Γ into Γ. Let α be an automorphism of $\mathcal{A}$. An algebraic ergodic system (Γ, U, φ) is an *algebraic model for* α if

$$J(U\gamma)=J(\gamma)^{\alpha} \qquad \text{for} \quad \gamma \in \Gamma,$$