187. On the Bi-ideals in Semigroups. II

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This note is a continuation of a recent paper of the author [8]. In this series some important results about bi-ideals of semigroups are summarized and some new results are announced. We adopt the standard notation and terminology due to A. H. Clifford and G. B. Preston [3].

Theorem 1. *Let S be a semigroup. Suppose that B is a bi-ideal, T is a subsemigroup of S, and the intersection $A=B\cap T$ is not empty. Then A is a bi-ideal of T.*

This is a consequence of a theorem concerning (m, n)-ideals (cf. the author [7], Theorem 1).

The following result shows that the existence of proper bi-ideal (in some cases) implies that of proper left (and right) ideal.

Theorem 2. *Suppose that A is a proper bi-ideal of a semigroup S, not being a left (right) ideal of S. Then the product BS (SB) is a proper right (left) ideal of S.*

The author proved the following statement [6].

Theorem 3. *Let S be a regular semigroup. Then every bi-ideal of S is a quasi-ideal, and conversely.*

K. M. Kapp [5] proved the following two results.

Theorem 4. *If S is a left simple semigroup, then every bi-ideal B of S is a right ideal.*

Theorem 5. *Let S be a semigroup with zero. If S is left 0-simple, then the sets of bi-ideals and quasi-ideals of S coincide.*

The following example shows that there exists such a bi-ideal which is not quasi-ideal.

Example 1. Let S be the semigroup of four elements 0, 1, 2, 3 with multiplication table

	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	1
3	0	0	1	2

.

It is easy to see that the subsemigroup $B=\{0, 2\}$ is a bi-ideal of S