201. On the Stability of Solutions of Certain Third Order Ordinary Differential Equations

By Tadayuki HARA
Osaka University

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1. Introduction. In this note we consider the asymptotic stability in the large of each of the zero solutions of the differential equations

$$\dddot{x}+a(t)\ddot{x}+b(t)\dot{x}+c(t)x=0 \qquad \left(\dot{x}=\frac{dx}{dt}\right), \tag{1.1}$$

$$\dddot{x}+a(t)f(x,\dot{x})\ddot{x}+b(t)g(x,\dot{x})\dot{x}+c(t)x=0, \tag{1.2}$$

where $a(t)$, $b(t)$ and $c(t)$ are positive and continuously differentiable functions on $[0,\infty)$ and $f(x,y)$, $f_x(x,y)$, $g(x,y)$ and $g_x(x,y)$ are continuous and real valued for all (x,y).

The zero solution of (1.1)(or (1.2)) is called asymptotically stable in the large, if it is stable and if every solution of (1.1) (or (1.2)) tends to zero as $t\to\infty$.

Many results have been obtained on the asymptotic property of autonomous equations of third order (cf. [1]).

In [2], K. E. Swick established conditions under which all the solutions of non-autonomous equations

$$\dddot{x}+p(t)\ddot{x}+q(t)g(\dot{x})+r(t)h(x)=0, \tag{1.3}$$

$$\dddot{x}+f(t,x,\dot{x})\ddot{x}+q(t)g(\dot{x})+r(t)h(x)=0, \tag{1.4}$$

tend to the zero solution as $t\to\infty$. We consider somewhat different non-autonomous equations (1.1) and (1.2) in which $a(t)$, $b(t)$ and $c(t)$ may oscillate to some extent.

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2. Theorems.

Theorem 1. *Suppose that $a(t)$, $b(t)$ and $c(t)$ are continuously differentiable on $[0,\infty)$ and following conditions are satisfied;*

(i) $A\geqq a(t)\geqq a_0>0$, $B\geqq b(t)\geqq b_0>0$, $C\geqq c(t)\geqq c_0>0$ *for* $t\in I=[0,\infty)$,

(ii) $a_0b_0-C>0$,

(iii) $\mu a'(t)+b'(t)-\dfrac{1}{\mu}c'(t)<\dfrac{a_0b_0-C}{2} \qquad \left(\mu=\dfrac{a_0b_0+C}{2b_0}\right)$,

(iv) $\int_0^\infty |c'(t)|\,dt<\infty$, $c'(t)\to 0$ as $t\to\infty$.

Then every solution $x(t)$ of (1.1) *is uniform-bounded and satisfies $x(t)\to 0$, $\dot{x}(t)\to 0$, $\ddot{x}(t)\to 0$ as $t\to\infty$.*