9. *Probabilities on Inheritance in Consanguineous Families. II*

By Yûsaku KOMATU and Han NISHIMIYA
Department of Mathematics, Tokyo Institute of Technology
(Comm. by T. FURUHATA, M.J.A., Jan. 12, 1954)

III. Simple mother-descendants combinations

1. Mother-child-νth descendant combination

We designate, in general, by

$$\pi_{\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) \equiv \bar{A}_{\alpha\beta}\kappa_{\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)$$

the probability of a combination consisting of an individual $A_{\alpha\beta}$ and its μth and νth collateral descendants $A_{\xi_1\eta_1}$ and $A_{\xi_2\eta_2}$, respectively, originated from the same spouse of $A_{\alpha\beta}$.

Three systems will be distinguished according to $\mu=\nu=1$, $\mu=1<\nu$ or $\mu>1=\nu$, and $\mu,\ \nu>1$. The *lowest system* has already been treated as the probability of mother-children combination[1]

$$\pi(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) \equiv \bar{A}_{\alpha\beta}\kappa(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) \qquad (\kappa\equiv\kappa_{11}).$$

In the present section we consider the *second system* while the last system will be postponed into the next section.

Now, based on an evident quasi-symmetry relation

$$\pi_{\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) = \pi_{\nu\mu}(\alpha\beta;\ \xi_2\eta_2,\ \xi_1\eta_1),$$

it suffices to deal with the former of the second system. The reduced probability $\kappa_{1\nu}$ is then defined by a recurrence equation

$$\kappa_{1\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) = \sum \kappa(\alpha\beta;\ \xi_1\eta_1,\ ab)\kappa_{\nu-1}(ab;\ \xi_2\eta_2).$$

It is shown that *the probability is expressed by the formula*

$$\kappa_{1\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) = \kappa(\alpha\beta;\ \xi_1\eta_1)\cdot\bar{A}_{\xi_2\eta_2} + 2^{-\nu}W(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2).$$

The quantity $W(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)$ in the residual term evidently vanishes out unless $A_{\xi_1\eta_1}$ possesses at least a gene in common with $A_{\alpha\beta}$, and its values are given as follows; cf. also a remark stated at the end of I, § 1:

$$\begin{aligned}
&W(ii;\ ii,\ ii) = 3i^2(1-i), && W(ii;\ ii,\ ig) = 3ig(1-2i),\\
&W(ii;\ ii,\ gg) = -3ig^2, && W(ii;\ ii,\ fg) = -6ifg,\\
&W(ii;\ ik,\ ii) = ik(2-3i), && W(ii;\ ik,\ ik) = k(i+2k-6ik),\\
&W(ii;\ ik,\ kk) = k^2(1-3k), && W(ii;\ ik,\ ig) = 2kg(1-3i),\\
&W(ii;\ ik,\ kg) = kg(1-6k), && W(ii;\ ik,\ gg) = -3kg^2,
\end{aligned}$$

1) Cf. a previous paper: IV. Mother-child combinations. **27** (1951), 587–620.