74. On Multiple Distributions

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Equations of evolution have been discussed by several authors, but it seems to me that researches have been done generally with respect to parametric operatorial equations or parametric distributional equations and scarcely with respect to proper distributional equations. So as a first step to the researches of the latter it will be of some interest to consider the general relations between parametric and proper distributional equations. To give a clarification of this relation we introduced the notion of multiple distributions defined in §3. At the same time the study of our multiple distributions will be helpful for the construction of resolvants of proper distributional equations.

Some other problems have also close relation to the study of multiple distributions, say, multiplication by particular distribution, the distributional treatment of Δ-type functions or of S-matrix. In this paper discussions of these problems are not stated, however, §5, §6, and §3 have relations to some parts of them.

1. First we modify a few B. H. Arnold's results. Let $S=\{\theta, x, y\ldots\}$ be a vector space over the real number field with zero vector θ, and $\mathfrak{B}$ be any collection of subsets of S satisfying the following axioms;

(B1) For any $x \in S$, $\{x\} \in \mathfrak{B}$.

(B2) The union of any two sets of $\mathfrak{B}$ is a set of $\mathfrak{B}$.

(B3) Any subset of a set of $\mathfrak{B}$ is a set of $\mathfrak{B}$.

(B4) Any scaler multiple of a set of $\mathfrak{B}$ is a set of $\mathfrak{B}$.

(B5) The convex hull of a set of $\mathfrak{B}$ is a set of $\mathfrak{B}$.

We call the elements of $\mathfrak{B}$ bounded subsets of the vector space S.

Definition 1. A subset G of S is called open if and only if whenever $g \in G$, there exists a convex set N such that for any $B \in \mathfrak{B}$ there exists a $\lambda > 0$ which satisfies $g+\lambda B \subset N \subset G$.

A set $T \subset S$ is called topologically bounded if and only if for each neighborhood U of θ there exists a λ with $T \subset \lambda U$. We denote by $\mathfrak{T}$ the collection of all subsets of S which are topologically bounded.

Lemma 1. *$\mathfrak{T} \supset \mathfrak{B}$, and the collection $\mathfrak{T}$ satisfies axioms from B1) to B5).*