121. *Some Trigonometrical Series. XVI*

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N. Wiener [1] proposed the problem to find the condition of the convergence of the series

$$\sum_{n=1}^{\infty} |s_n(x)-f(x)|, \tag{1}$$

and

$$\sum_{n=1}^{\infty} (s_n(x)-f(x))^2, \tag{2}$$

where $s_n(x)$ is the nth partial sum of Fourier series of $f(x)$. The uniform convergence of (2) was treated in [2].

The object of this paper is to find the condition of almost everywhere convergence of the series

$$\sum_{n=1}^{\infty} |s_n(x)-f(x)|^{\lambda}. \tag{3}$$

In the case $\lambda=1$, that is, in (1) T. Tsuchikura [3] has gotten the condition by the Fourier coefficients of $f(x)$. We prove the following

Theorem. *Let $p>1$, $p\geqq\lambda\geqq1$, and ε be any positive number. If $f(x)$ is of the power series type*[1] *and*

$$\left(\int_0^{2\pi} |f(x+t)-f(x)|^p\,dx\right)^{1/p} \leqq At^{1/\lambda}\Big/\left(\log\frac{1}{t}\right)^{(1+\varepsilon)/\lambda} \tag{4}$$

then the series (3) converges almost everywhere.

In the proof we use the technic due to A. Zygmund [4] and his lemma:

Lemma. *Suppose that $p>1$ and*

$$\Big\|\sum_{\nu=m}^{n} \gamma_\nu e^{i\nu x}\Big\|_p \leqq C$$

where $\|\ \|_p$ denotes the L^p-norm and suppose that

$$|\lambda_\nu| \leqq M,\ \sum_{\nu=m}^{n-1} |\lambda_\nu-\lambda_{\nu+1}| \leqq M,$$

then

$$\Big\|\sum_{\nu=m}^{n} \gamma_\nu\lambda_\nu e^{i\nu x}\Big\|_p \leqq A_pMC.$$

Let us now prove the theorem. It is sufficient to prove that the series

$$\sum_{n=1}^{\infty} \int_0^{2\pi} |s_n(x)-f(x)|^{\lambda}\,dx \tag{5}$$

is convergent. Then

1) This condition is implied by (4) when $\lambda>1$.