147. On the Property of Lebesgue in Uniform Spaces. V

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In a series of my Note [5], we studied the uniform spaces having Lebesgue property for finite covering. In this Note, we shall study the uniform space with Lebesgue property for any covering. Such a space was also studied by S. Kasahara [6, 7], one of my colleagues, and he obtained some important results.

First, we shall consider a uniform space with a unique structure.

Let S be a uniform space with a unique structure and S^* a compactification of S. This is possible by P. Samuel [8]. The unique structure of the compact space S^* induces on S a uniform structure. Therefore, the original structure on S coincides with the induced structure. Hence S is precompact. On the other hand, there is a uniform structure on S for which every continuous function is uniformly continuous (A. Weil [9], p. 16). Hence, if S is metrisable, S has the Lebesgue property, and this shows that S is compact. Further, if S has the Lebesgue property, then S is compact, since a theorem of my Note ([5], p. 441).

Therefore, we have

Theorem 1. If a uniform space with a unique structure has the Lebesgue property, then it is compact.

If E is a uniform space, there is a strongest uniform structure on E compatible with the original structure. We say such a structure the universal uniform structure. J. Dieudonné [3] proved the following

Theorem. If E is paracompact, the filter of neighbourhoods of the diagonal in $E\times E$ is the filter of surroundings of the universal structure of E.

On the other hand, P. Samuel [8] proved the following

Theorem. If every continuous mapping of S onto any uniform space is uniformly continuous, then the uniform structure of S is universal.

Theorem 2. If any covering of a uniform space S has the Lebesgue property, then the uniform structure of S is universal.

Proof. We shall prove that every continuous mapping onto a uniform space is uniformly continuous. By the theorem mentioned above, we have the conclusion of Theorem 2. Let F be a given uniform space, and $f(x)$ a continuous mapping of E onto F. For