4. On Images of an Open Interval under Closed Continuous Mappings

By Kiiti MORITA

Tokyo University of Education, Tokyo

(Comm. by K. KUNUGI, M.J.A., Jan. 12, 1956)

1. Introduction. A mapping of a topological space X onto another topological space Y is said to be closed if the image of every closed subset of X is closed in Y. As is well known, in order that a metric space Y be the image of the closed line interval $[0,1]$ under a closed continuous mapping it is necessary and sufficient that Y be a locally connected continuum.

In the present note we shall establish the following theorem, which is an analogue of the celebrated theorem of Hahn-Mazurkiewicz mentioned above and may be considered as a generalization of it since any closed continuous mapping of the open line interval $(0,1)$ onto a locally connected continuum can be extended over $[0,1]$ by our Theorem 3 below.

Theorem 1. *In order that a metric space Y be the image of the open line interval* $(0,1)$ *under a closed continuous mapping it is necessary and sufficient that Y be a separable, locally compact, connected, locally connected space with at most two end-points in the sense of Freudenthal (i.e. $\gamma(Y)-Y$ consists of at most two points).*

Here $\gamma(Y)$ means the Freudenthal compactification of the space Y (cf. [1], [2]).

For any positive integer m let Q_m be the union of m closed segments a_ia_0, $i=1,2,\cdots,m$, each two having only one point a_0 in common, and let P_m be the space obtained from Q_m by removing the points a_i, $i=1,2,\cdots,m$. Then P_1 and P_2 are homeomorphic to $(0,1]$ and $(0,1)$ respectively, and hence Theorem 1 is contained in the following theorem.

Theorem 2. *In order that a metric space Y be the image of P_m under a closed continuous mapping it is necessary and sufficient that Y be a separable, locally compact, connected, locally connected space with at most m end-points in the sense of Freudenthal.*

2. Lemmas. We shall first prove

Lemma 1. *Let f be a closed continuous mapping of a metric space X onto another metric space Y. If A is a closed subset of Y whose boundary $\mathfrak{B}A$ is compact, then $\mathfrak{B}f^{-1}(A)$ is also compact.*

Proof. Let us put

$$V_i=A\smile\{y\mid\rho(y,\mathfrak{B}A)<1/i\},$$