66. On Weakly Compact Regular Spaces. II

By Shouro KASAHARA
Kobe University
(Comm. by K. KUNUGI, M.J.A., May 15, 1957)

In their paper [4], S. Mardešić and P. Papić have given interesting characterizations of pseudo-compact spaces,[1] and their results lead us to study *weakly compact spaces*[2] introduced by them. The main object of this paper is to give some characterization of weakly compact regular spaces. By a theorem of S. Mardešić and P. Papić, the results stated below give characterizations of pseudo-compact spaces whenever the spaces considered are completely regular.

A topological space[3] E is said to be *weakly compact* if to every pairwise disjoint infinite family of open sets O_α of E there corresponds a point $x \in E$ such that each neighbourhood V of x meets infinitely many O_α. A family of subsets of a topological space E is said to be *locally finite* if each point of E possesses a neighbourhood which meets at most a finite number of the members of the family.

It is known[4] that a completely regular space is pseudo-compact if and only if every locally finite open covering of it has a finite subcovering, or equivalently, every star finite open covering of it has a finite subcovering. This proposition is justified by the following

THEOREM 1. *The following properties of a regular space E are equivalent:*

(1) *E is weakly compact.*

(2) *Every infinite open covering of E has a proper subfamily whose union is dense in E.*

(3) *Every locally finite family $\{O_\alpha\}$ of open sets of E has a finite subfamily whose union contains every O_α.*

(4) *Every locally finite open covering of E has a finite subcovering.*

(5) *Every locally finite open covering of E has a finite subfamily whose union is dense in E.*

(6) *Every star finite open covering of E has a finite subcovering.*[5]

1) A completely regular space is said to be *pseudo-compact* if every continuous function on the space is bounded.

2) "Espaces faiblement compacts". See S. Mardešić and P. Papić [4].

3) Throughout this paper we assume that every topological space satisfies the axiom T_1.

4) See for example K. Iséki and S. Kasahara [2].

5) A covering $\{O_\alpha\}$ of a space is termed *star finite* if each member of $\{O_\alpha\}$ meets only a finite number of O_α's.