7. *Ideals in Semirings*

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The theory of semiring was first developed by H. S. Vandiver and he has obtained important results of the subjects. Recently, the study of the theory of semiring was made by S. Bourne [1], H. Zassenhaus [2] and the present author [3].

Let S be a semiring (for the definition of a semiring and ideals, see S. Bourne [1]). An ideal P of S is *prime*, if and only if $AB\subset P$ for two ideals A, B implies $A\subset P$ or $B\subset P$. In this paper [3], the present author has given some characterisations of prime ideals. First, we shall show some new criteria for prime ideals.

Theorem 1. If P is an ideal in the semiring S, then the following propositions are equivalent:

(1) *P is prime ideal.*

(2) *If (a), (b) are principal ideals, and $(a)(b)\in P$, then $a\in P$ or $b\in P$.*

(3) *$aSb\subset P$ implies $a\in P$ or $b\in P$.*

(4) *If R_1, R_2 are right ideals and $R_1R_2\subset P$, then $R_1\subset P$ or $R_2\subset P$.*

(5) *If L_1, L_2 are left ideals and $L_1L_2\subset P$, then $L_1\subset P$ or $L_2\subset P$.*

(6) *If R and L are right and left ideals respectively in S such that $RL\subset P$, then $R\subset P$ or $L\subset P$.*

(7) *If $(a)(b)\subset P$, then $a\in P$ or $B\in P$.*[*]

(8) *If $(a)_r(b)_r\subset P$, then $a\in P$ or $b\in P$.*

(9) *If $(a)_l(b)_l\subset P$, then $a\in P$ or $b\in P$.*

In [3], we proved the equivalence of propositions (1), (2), (3), (4) and (5). The implications (3)→(4)→(8)→(2), (3)→(5)→(9)→(2), (6)→(7)→(2) and (2)→(3) are trivial. Therefore we shall show (3)→(6). Let R be a right ideal, and L a left ideal. Suppose that $RL\subset P$ and R is not in P. Then there is an element a in R such that $a\notin P$. Hence, for each element b of L, we have

$$aSb\subseteq RL\subseteq P.$$

Therefore, proposition (3) implies $b\in P$, and this shows that $L\subset P$. This completes the proof of Theorem 1.

We can prove a theorem of completely prime ideal in semiring which is similar to a result of O. Steinfeld [5].

An ideal P is *completely prime* if and only if, $ab\in P$ for a, b in S implies $a\in P$ or $b\in P$. Then we have the following

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[*] $(a)_r$ is the principal right ideal generated by a: $\{aS+na\mid(n=1,2,\cdots), s\in S\}$, and $(b)_l$ is the principal left ideal generated by b.