

26. Note on Idempotent Semigroups. III

By Naoki KIMURA

Tokyo Institute of Technology and Tulane University

(Comm. by K. SHODA, M.J.A., Feb. 12, 1958)

§ 1. This note is an abstract of the paper (Naoki Kimura [1]), in which the author proved the structure theorems of some special idempotent semigroups. Terminologies in the previous papers (Kimura [2] and Miyuki Yamada and Kimura [3]) will be used without definitions.

§ 2. An idempotent semigroup is called

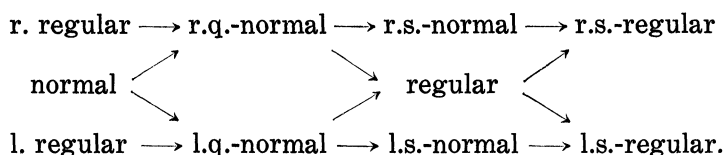
(1) *right (left) semi-regular* if $axy = axyayxy$ ($xya = xyxaxya$),

(2) *right (left) semi-normal* if $axy = axyay$ ($xya = xaxya$),

(3) *right (left) quasi-normal* if $axy = axay$ ($xya = xayay$),

for all a, x, y .

Then the following implications are easy to prove:



Further, we have the following lemmas:

LEMMA 1. *An idempotent semigroup is regular if and only if it is both left and right semi-regular.*

LEMMA 2. *An idempotent semigroup is normal if and only if it is both left and right quasi-normal (semi-normal).*

§ 3. Let $\mathfrak{P}(\mathfrak{Q})$ be the equivalence on an idempotent semigroup S defined by

$x\mathfrak{P}y$ if and only if $xy = y$ and $yx = x$,

$x\mathfrak{Q}y$ if and only if $xy = x$ and $yx = y$.

Then we have the following representation theorems.

THEOREM 1. $\mathfrak{P}(\mathfrak{Q})$ is a congruence on an idempotent semigroup S if and only if S is left (right) semi-regular. Further, in this case the quotient semigroup $S/\mathfrak{P}(S/\mathfrak{Q})$ is left (right) regular.

From this theorem and Lemma 1, we have

COROLLARY. *Both \mathfrak{P} and \mathfrak{Q} are congruences on an idempotent semigroup S if and only if S is regular. Further, in this case S is isomorphic to the spined product of S/\mathfrak{P} and S/\mathfrak{Q} with respect to its structure semilattice.*

REMARK. This corollary is essentially the same as Theorem 2 in [2], and the above method gives an alternative proof for it.