73. On Some Existence Theorems on Multiplicative Systems. II. Maximal Subsystems*)

By Naoki KIMURA

Tokyo Institute of Technology and Tulane University of Louisiana

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§ 1. Introduction. We have given several existence theorems on multiplicative systems in the previous paper [1]. Here we shall be concerned in this short note with a part which shares a portion on one side of duality whose other side has been the main part of [1]. We shall use terminologies defined in [1] without mentioning.

In the second section we shall state existence theorems of the greatest subsystem, while in the fourth section we shall give those of maximal subsystems. Before presenting the latter we need to introduce the notion of amalgamated product, which appears new in general case and will be defined in the third section.

The fifth section is devoted to make a comparison between the notions defined and used in this note and those in [1]. The dual part of § 6 in the previous paper [1] is omitted here which will be given in the subsequent paper.

In this paper the empty set is considered as a system, unless otherwise specified. Thus for any system, the empty system is a subsystem of it. Also systems discussed in this paper are assumed to be multiplicative systems with the same set M of multiplications. So for brevity we do not mention M in what follows.

§ 2. Existence of the greatest P-system. Let P be a property on systems and let S be a system. Then a subsystem T of S is called the *greatest* P*-subsystem*, if it satisfies the following conditions:

(1) T is a P-system,

(2) if T' is a subsystem of S which satisfies P, then $T' \subset T$.

A property P is called *regular* (*preregular*), if for any system (at least one subsystem of which satisfies P), there exists its greatest P-subsystem.

Let $\{S_k: k \in K\}$ be a family of systems. We shall denote the free product of it by $S^* = \Pi^*\{S_k: k \in K\}$. Then we have the natural imbedding $i_k: S_k \to S^*$ by which we can regard S_k as a subsystem of S^*. Any quotient T of S^* under an onto homomorphism $h: S^* \to T$ is called a *semi-free product* if h sends each S_k into T in the one-to-one

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