120. A Generalization of Vainberg's Theorem. I

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(Comm. by K. KUNUGI, M.J.A., Oct. 13, 1958)

1. Let E be a measurable set in Euclidean n-space and $f(u, t)$ be a real valued function defined for u real and t in E such that it is *continuous* as a function of u for almost all $t \in E$ and *measurable* as a function of t for all u.

By this function $f(u, t)$ we define for every real valued measurable function $x(t)$

(1.1) $$\mathfrak{H}(x(t)) = f(x(t), t).$$

Then $\mathfrak{H}(x(t))$ is also measurable function on E and $\mathfrak{H}$ establishes a transformation on the space of measurable functions on E into itself.

Recently in [2] M. M. Vainberg proved that *in order that* $\mathfrak{H} \equiv f(u, t)$ *maps* $L_p(E)$ *into* $L_{p_1}(E)$ $(p, p_1 > 0)$ *it is necessary and sufficient that there exist a positive number* γ *and a function* $a(t)$ *belonging to* $L_{p_1}(E)$ *such that*

(1.2) $$|f(u, t)| \leq a(t) + \gamma |u|^{\frac{p}{p_1}}$$

for all $t \in E$, $u \in (-\infty, +\infty)$.

Let B be a Banach space consisting of measurable functions on E and B^* is its *conjugate* space. The operators $\mathfrak{H} \equiv f(u, t)$ which map B into B^* are particularly interesting and discussed by several authors, because of their connection to the theory of non-linear integral operators of the form:

(1.3) $$Ax(t) = \int_E K(t, s) f(x(s), s) ds.$$

We shall generalize the Vainberg's Theorem on *modulared semi-ordered linear spaces* and point out that $\mathfrak{H}$ is characterized by *conjugately similar correspondences*[1)] [1, § 59], in the case that $\mathfrak{H}$ operates into the conjugate spaces. Here we shall prove only the fundamental theorem, which allows to obtain the Vainberg's Theorem in more general form. For want of space the details will be discussed in the following paper.

2. Let R be a *modulared semi-ordered linear space*,[2)] and $m(a)$ $(a \in R)$ be *a modular* on R. The totality of all elements $a \in R$ such that

1) The definition of the conjugately similar correspondence will be stated in the following paper.

2) We suppose that semi-ordered linear space is always universally continuous in the sequel, i.e. $a_\lambda \geq 0$ $(\lambda \in \Lambda)$ implies $\bigcap_{\lambda \in \Lambda} a_\lambda \in R$. The notations and terminologies used here are the same ones used in [1].