119. *On Semi-continuity of Functionals. I*

By Shōzō KOSHI

Mathematical Institute, Hokkaidō University, Sapporo

(Comm. by K. KUNUGI, M.J.A., Oct. 13, 1958)

1. *Introduction.* H. Nakano obtained the following decomposition theorem (cf. [3]). Let R be a *universally continuous semi-ordered linear space* (*a conditionally complete vector lattice* in Birkhoff's terminology [2]). A linear functional f on R is said to be *bounded* if $\sup_{|b|\leq|a|}|f(b)|<+\infty$ for every $a\in R$.

A bounded linear functional f is decomposed into two linear functionals f_1, f_2 such that

(1.1) $$f=f_1+f_2,$$

f_1 is a universally continuous linear functional[1] *and f_2 is orthogonal to every universally continuous linear functional.*

The structure of f_1 was fully discussed by Nakano [3]. But f_2 is not clear yet. On the other hand, he considered totally continuous spaces.[2]

Recently I. Amemiya conjectured that *if R is totally continuous, then f_2 is 0 on some complete semi-normal manifold of R.* (A semi-normal manifold (1-ideal in Birkhoff's terminology) $M\subset R$ is said to be *complete* if $|a|\frown|b|=0$, $a\in R$ (for every $b\in M$) implies $a=0$. In this case, it is proved that for any $a\in R$, there exist $a_\lambda\in M(\lambda\in\Lambda)$ with $a=\bigcup_{\lambda\in\Lambda}a_\lambda$ [3].)

The aim of this note is to prove Theorem 1 which deduces the Amemiya's conjecture as a special case. In the sequel, we will use the notations and terminologies in [3].

2. *Definitions and lemmas.* According to Nakano [3], R is said to be *totally continuous* if *for any double sequence of projectors* $[p_{i,j}]\uparrow_{j=1}^{\infty}[p]$, *there exists a sequence of projectors* $[p_k]\uparrow_{k=1}^{\infty}[p]$ *and integers* $l(i,k)$ $(i,k=1,2,\cdots)$ *with* $[p_k]\leqq[p_{i,l(i,k)}]$.

I. Amemiya proved that if the hypothesis of continuum is satisfied, then a universally continuous and totally continuous semi-ordered linear space is *super-universally continuous*, that is, for any *system* $a_\lambda\geqq0$ $(\lambda\in\Lambda)$, there exist countable $\lambda_i\in\Lambda$, a_{λ_i} $(i=1,2,\cdots)$ with $\bigcap_{\lambda\in\Lambda}a_\lambda=\bigcap_{i=1}^{\infty}a_{\lambda_i}$ [1]. If R is super-universally continuous and totally continuous, it is proved

1) $a_\lambda\downarrow_{\lambda\in\Lambda}0$ implies $\inf_{\lambda\in\Lambda}|f(a_\lambda)|=0$.

2) The space of measurable functions on measure space which is σ-finite is an example of totally continuous space.