114. *On Certain Examples of the Crossed Product of Finite Factors. I*

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The purpose of the present note is two folds: The first is to establish that the hyperfinite continuous factor has many groups of outer automorphisms and the second is to show that the crossed product of the hyperfinite continuous factor is not always hyperfinite, which may give a right to the abstract treatment of the crossed products of von Neumann algebras. As stated in Turumaru [5] or in the previous [2], the method of the factor construction due to Murray-von Neumann [1] is nothing but the crossed product of an abelian algebra. It may be expected that the crossed product construction on non-abelian algebras is useful in the future study of von Neumann algebras. The present note may be observed as the first step towards it.

We utilize Murray-von Neumann's notation calculation as possible to save the space for description. The terminology is the same as in the previous [2]. Thus, for example, every automorphism in this paper means always a $*$-automorphism of a von Neumann algebra acting on a separable Hilbert space.

1. **Hyperfinite continuous factor and its automorphism group.** The pair of numbers $\{0, 1\}$ becomes an additive group by mod. 2. Its Haar measure μ is such that $\mu(\{0\})=\frac{1}{2}$, $\mu(\{1\})=\frac{1}{2}$. For every element g of an enumerably infinite group G, we associate the group $E_g=\{0, 1\}$ and its Haar measure μ_g. By X denotes the product space of $E_g(g\in G)$ and the integration on X means always the one by the product measure of μ_g. That is, X is the set of all systems $x=[x_g;\ g\in G]$ where each $x_g=0$ or 1 and the total measure of X is one. Next put Γ the set of those $x=[x_g;\ g\in G]$ for which $x_g=0$ except for a finite number of g's. By the group operation of each component, Γ becomes a group, i.e.

$$\gamma+\gamma'=[\gamma_g+\gamma'_g] \quad (\text{mod. } 2) \quad \text{for } \gamma=[\gamma_g],\ \gamma'=[\gamma'_g].$$

Furthermore, defining

$$xT_\gamma=x+\gamma=[x_g+\gamma_g] \quad \text{for} \quad x=[x_g]\in X,$$

we get a measure preserving, ergodic transformation group $\{T_\gamma \mid \gamma\in\Gamma\}$ on X [3, Lemma 7.5.1]. As easily seen, this $\{T_\gamma\}$ is isomorphic to Γ