78. *Cauchy Integral on Riemann Surfaces*

By Kôichi TARUMOTO

(Comm. by K. KUNUGI, M.J.A., July 13, 1959)

In recent times many authors treated the linear functional methods in the theory of functions of a complex variable. In these studies it is of great use to study the behaviour of the considered function on the boundary of the domain.

In this paper we will study the boundary behaviour of the Cauchy integral on a Riemann surface.

1. Let $\mathfrak{B}$ be a compact or non-compact subregion with finite relative boundaries on a non-compact Riemann surface. We can recognize the existence of a differential $dN(\mathfrak{P}_\zeta, \mathfrak{P}_z)=A(\zeta, z)d\zeta$ with the following properties (Elementardifferential of $\mathfrak{R}$ in the terminology of Behnke-Stein).[1)]

$A(\zeta, z)$ is defined in a cylinder region $\mathfrak{R}_{\mathfrak{P}_\zeta}\times\mathfrak{R}_{\mathfrak{P}_z}$, and

1. $A(\zeta, z)$ is meromorphic with respect to both arguments.
2. If $\mathfrak{P}_\zeta\neq\mathfrak{P}_z$, $dN(\mathfrak{P}_\zeta, \mathfrak{P}_z)$ is finite.
3. If $\mathfrak{P}_\zeta=\mathfrak{P}_z$, $dN(\mathfrak{P}_\zeta, \mathfrak{P}_z)$ has a pole with residue 1 at $\mathfrak{P}_z$.

When $f(\mathfrak{P}_\zeta)$ is a continuous function on the relative boundary Γ of $\mathfrak{B}$ (where $\Gamma=\bigcup_{j=1}^{p}\gamma_j$, and γ_j is an analytic Jordan curve), and $dN(\mathfrak{P}_\zeta, \mathfrak{P}_z)$ is an Elementardifferential of $\mathfrak{R}$, the integral $F(\mathfrak{P}_z)=\frac{1}{2\pi i}\int_\Gamma f(\mathfrak{P}_\zeta)\cdot dN(\mathfrak{P}_\zeta, \mathfrak{P}_z)$ may be called the Cauchy integral of $\mathfrak{B}$. Each of the integral $\frac{1}{2\pi i}\int_{\gamma_j} f_j(\mathfrak{P}_\zeta)dN(\mathfrak{P}_\zeta, \mathfrak{P}_z)$, where $f_j(\mathfrak{P}_\zeta)$ is the restriction of $f(\mathfrak{P}_\zeta)$ on γ_j, is called the j-th component of $F(\mathfrak{P}_z)$ and denoted by $F_j(\mathfrak{P}_z)$.

In the sequel it is fundamentally important that for each j $(j=1, 2,\cdots, p)$ there exist

1) a strip $\mathfrak{R}_j$ of $\mathfrak{R}$ containing γ_j,

2) an annulus $r_1<x<r_2$ in the complex plane with $r_1<1<r_2$,

3) a one-to-one conformal mapping of the strip $\mathfrak{R}_j$ into the annulus such that γ_j is mapped on the circumference of the unit circle.

The strip and the associated mapping are not in any way unique, but in our study it will sometimes be convenient to fix one of them and denote it by $\mathfrak{R}_j$ and $\mathfrak{P}_\zeta=\lambda_j(t)$. Given a function $f(\mathfrak{P}_z)$ on $\mathfrak{R}$ and

1) Cf. H. Behnke und F. Sommer: Theorie der analytischen Funktionen einer komplexen Veränderlichen, 555–559 (1955).