34. A Characteristic Property of L_p-Spaces $(p>1)$. II

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In the previous paper,[1] we gave a characteristic property of L_p-spaces $(p>1)$. The purpose of this paper is to give another characterization.

In the case of L_p $(p>1)$, the transformation

(1) $$Tx(t)=|x(t)|^{p-1}\cdot\operatorname{sgn} x(t)$$

is a one-to-one correspondence between L_p and L_q $(q=p/p-1)$, and the functional (called a modular)

(2) $$m(x)=\int_0^1(T\xi x, x)\,d\xi=\frac{1}{p}\int_0^1|x(t)|^p\,dt$$

is well defined. Putting

(3) $$\|x\|=\inf_{m(\xi x)\leqq 1}\frac{1}{|\xi|},$$

we get a norm of L_p and

$$\|x\|=\left(\frac{1}{p}\int_0^1|x(t)|^p\,dt\right)^{\frac{1}{p}} \qquad (x\in L_p).$$

The conjugate norm of it is

(4) $$\|\bar{x}\|=\sup_{\|x\|\leqq 1}|(\bar{x}, x)|=p^{\frac{1}{p}}\left(\int_0^1|\bar{x}(t)|^q\,dt\right)^{\frac{1}{q}} \qquad (\bar{x}\in L_q).$$

Then, it is easily seen that the transformation (1) is *norm-preserving*:

$$\|x\|=\|y\| \text{ in } L_p \text{ implies } \|Tx\|=\|Ty\| \text{ in } L_q.$$

In this paper, we will prove that this property of T is characteristic for L_p $(p>1)$ among such Banach spaces that have some transformations like (1), namely, conjugately similar spaces.

Definition. A universally continuous semi-ordered linear space R is said to be *conjugately similar*,[2] if R is reflexive and there exists a one-to-one transformation T from R onto its conjugate space $\bar{R}$ with the following properties:

(i) $T(-a)=-Ta$ $(a\in R)$;
(ii) $Ta\leqq Tb$ if and only if $a\leqq b$ $(a, b\in R)$;
(iii) $(Ta, a)=0$ implies $a=0$.

The above transformation T is called a *conjugately similar correspondence*.

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1) K. Honda and S. Yamamuro [1].
2) Throughout this paper, notations and terminologies are according to H. Nakano [2].