# 51. A Remark on a Paper of Greub and Rheinboldt

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1. In the first place, it will be shown by an elementary inspection the following

THEOREM 1. *For* $0<m<M$, *the following inequality holds true;*

$$\int_m^M t\,d\mu(t)\cdot\int_m^M \frac{1}{t}\,d\mu(t)\leqq\frac{(M+m)^2}{4Mm}, \tag{1}$$

*for any positive Stieltjes measure* $\mu$ *on* $[m, M]$ *with* $\|\mu\|=1$.

Consider a line-segment $C$ and a curve $D$ figured in $(t, s)$-plane by $(t, t)$ and $\left(t, \frac{1}{t}\right)$ respectively (for $m\leqq t\leqq M$). Putting

$$d=\int_m^M t\,d\mu(t),\qquad e=\int_m^M \frac{1}{t}\,d\mu(t),$$

$(d, d)$ is the centre of gravity of $C$ weighted by $\mu$, and $(d, e)$ is of $D$ weighted by the same $\mu$. Clearly, $(d, d)$ lies on $C$, and $(d, e)$ lies in the bow shaped territory bounded below by $D$ and above by its string connected $(m, 1/m)$ and $(M, 1/M)$ or the line figured by $(t, g(t))$ where

$$g(t)=\frac{(M+m)-t}{Mm}.$$

It is now obvious that the left hand side of (1), say $c$, is the product of the $s$-coordinates of two centres of gravity. Hence $(d, c)$ lies below a curve figured by $(t, h(t))$ with

$$h(t)=t\,g(t)=\frac{(M+m)t-t^2}{Mm}.$$

Therefore, $c$ amounts its maximum, if possible, when

$$Mm\,h'(t)=(M+m)-2t=0,$$

or $t=(M+m)/2$. Thus,

$$c\leqq h\left(\frac{M+m}{2}\right)=\frac{(M+m)^2}{4Mm},$$

which proves (1).

Incidentally, it is obvious that $c$ attains its maximum when

$$\mu(\{m\})=\mu(\{M\})=\frac{1}{2}.$$

THEOREM 2. *If $f$ is a continuous function defined on a compact set satisfying*

$$0<m\leqq f(x)\leqq M, \tag{2}$$

*then*