49. *On Locally Compact Halfrings*

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1. Introduction. *Definition* 1. A halfring H is a semiring which can be embedded in a ring.

Since we shall confine ourselves to a semiring with commutative addition, it is necessary and sufficient that the cancellation law of addition holds in this semiring for it to be a halfring.

The product set $H\times H$ forms again a halfring according to the laws

$$(1)\qquad \begin{aligned}(i_1, j_1)+(i_2, j_2)&=(i_1+i_2, j_1+j_2),\\ (i_1, j_1)(i_2, j_2)&=(i_1i_2+j_1j_2, i_1j_2+j_1i_2).\end{aligned}$$

The diagonal $\Delta=\{(x, x)|x\in H\}$ of H is a two-sided ideal in $H\times H$.

Definition 2. Two elements (i_1, j_1), (i_2, j_2) of the halfring $H\times H$ are said to be equivalent modulo Δ, if there exist elements (x, x) and (y, y) in Δ such that

$$(2)\qquad (i_1, j_1)+(x, x)=(i_2, j_2)+(y, y).$$

This equivalence relation is a special case of the one given in a previous paper [1]. From (2) we obtain that $i_1+x=i_2+y$, $j_1+x=j_2+y$, $i_1+x+j_2+y=i_2+y+j_1+x$ and $i_1+j_2=i_2+j_1$. Also, if $i_1+j_2=i_2+j_1$ then $(i_1, j_1)+(j_2, j_2)=(i_2, j_2)+(j_1, j_1)$ and $(i_1, j_1)\sim(i_2, j_2)$. The difference ring $R=H\times H/\Delta$ is defined to be the ring generated by H. Let ν denote the natural homomorphism of $H\times H$ onto R, then the halfring H is embedded in the ring R, for the mapping $h\rightarrow\nu(h+a, a)$, for any a, is an isomorphism of H into R.

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2. Quotient spaces. *Definition* 3. A topological halfring is a halfring H together with a Hausdorff topology on H under which the halfring operations are continuous.

We introduce in R the quotient topology, that is the largest topology for R such that the projection (quotient map) ν is a continuous mapping of $H\times H$ onto R. We assume that H is a locally compact space, then $H\times H$ is locally compact in the product topology. If the projection ν is open, that is the image of each open set is open, then R is also a locally compact space [5]. Hence, it will be fruitful to impose a property on H which will insure that ν be an open mapping. Furthermore, if ν is open then R is a topological ring [3]. In a recent paper [6], N. J. Rothman imposes such a topological and