21. On the Extension Theorem of the Galois Theory for Finite Factors

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1. We have shown that the fundamental theorem of the Galois theory remains true for finite factors [3] as same as for simple Noetherian rings. Subsequently, in this note, we shall discuss about the so-called extension theorem[1] for finite factors.

We denote by A a continuous finite factor standardly acting on a separable Hilbert space H and by G a finite group of outer automorphisms of A. Put B the set of all elements invariant by G. B is a subfactor of A. Now let C and D be two intermediate subfactors between A and B, then by the fundamental theorem of the Galois theory, there correspond the Galois groups E and F for C and D respectively. That is, E and F are subgroups of G by which C and D are shown as the sets of elements invariant by E and F respectively. Then we may give the extension theorem in the following form.

THEOREM. *Let σ be an isomorphism between C and D fixing every elements of B, then σ may be always extended to an automorphism of A which belongs to G.*

2. We shall begin with some preliminaries. By A^θ we mean the set A equipped with the inner product $\langle a^\theta \mid b^\theta\rangle=\tau(ab^*)$ defined by the standard trace τ of A. As well known, A is faithfully represented on the completion Hilbert space of A^θ. The representation is spatially isomorphic to A acting on H, whence we may identify the representation with A and so A^θ with a dense subset of H. Thus $1^\theta \in H$ gives a trace element of A. The subspace $[1^\theta C]$[2] of H belongs to C'. Since $C'\subset B'$ it belongs B' too. Hence its relative dimension $\dim_{B'}[1^\theta C]$ with respect to B' is meaningful.

As well known, the automorphism group G permits a unitary representation $\{u_g\}$ on H such that $x^g=u_g^* x u_g$ for $x\in A$. Furthermore, as shown in [3], putting $x'^g=u_g^* x' u_g$ for $x'\in A'$, G can be seen as a group of outer automorphisms of A'. Hence we may construct the crossed product $G\otimes A'$ of A' by G, cf. [2]. This can be understand as a von Neumann algebra acting on a Hilbert space $\boldsymbol{H}$ composed of all functions defined on G taking values in H. We show by $\sum_g g\otimes\varphi_g$ a function belonging to $\boldsymbol{H}$ which takes value φ_g at $g\in G$. Then $a'\in A'$

1) Refer to [5] for the theorem of rings with the minimum condition.

2) $[1^\theta C]$ means the metric closure of the set $\{1^\theta c \mid c\in C\}$.