# 50. Remarks on Katětov's Uniformly 0-dimensional Mappings

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It seems to me that the notion of uniformly 0-dimensional mappings introduced by M. Katětov plays an essential rôle in his dimension theory for non-separable metric spaces [3]. Let $R$ and $S$ be metric spaces (with the metric $\rho_1$ and $\rho_2$ respectively) and $f$ a continuous mapping of $R$ into $S$. According to him, $f$ is called $((\rho_1, \rho_2)$-) uniformly 0-dimensional if the following condition is satisfied.

(*) For any $\varepsilon>0$ there exists a $\delta>0$ such that when $M\subset S$ and dia $M$[1] $<\delta$, $f^{-1}(M)$ can be decomposed into mutually disjoint relatively open (in $f^{-1}(M)$) sets whose diameters are less than $\varepsilon$.

He proved that for any metric space $R$ with $\dim R$[2] $\leq n$[3] there exists a uniformly 0-dimensional continuous mapping of $R$ into the Euclidean $n$-space $E^n$. With the aid of this fundamental theorem he proved the decomposition theorem and in consequence the equality $\dim R=\operatorname{Ind} R$[4] for metric space $R$. Modifying Katětov's definition, we shall give in this note a definition of uniformly 0-dimensional continuous mappings of normal spaces into normal ones. Let $R$ and $S$ be normal spaces and $f$ a uniformly 0-dimensional continuous mapping, in our sense, of $R$ into $S$. Then it is the main purpose to show that $\dim R\leq\dim S$ and $\operatorname{Ind} R\leq\operatorname{Ind} S$.

**Definition.** Let $R$ and $S$ be topological spaces. Let $U=\{\mathfrak{U}_\lambda;\ \lambda\in\Lambda\}$ and $V=\{\mathfrak{V}_\mu;\ \mu\in M\}$ be respectively collections of open coverings of $R$ and $S$. Let $f$ be a continuous mapping of $R$ into $S$. Then we call that $f$ is $(U, V)$-*uniformly* 0-*dimensional* if the following condition is satisfied:

(**) For any $\lambda\in\Lambda$ there exists a $\mu\in M$ such that for any $V\in\mathfrak{V}_\mu$ there exists a collection $\{H_\alpha;\ \alpha\in A\}$ of disjoint open sets of $R$ with $\cup\{H_\alpha;\ \alpha\in A\}=f^{-1}(V)$ which refines $\mathfrak{U}_\lambda$.

Throughout this note the following notations will be used.

$U_F$ = *the collection of all finite open coverings of* $R$.

$U_B$ = *the collection of all binary open coverings*[5] *of* $R$.

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1) dia $M$ denotes the diameter of $M$.

2) $\dim R$ denotes the covering dimension of $R$.

3) Throughout this note $n$ denotes a non-negative integer.

4) $\operatorname{Ind} R$ denotes the large inductive dimension of $R$ defined inductively as follows. For the empty set $\phi$ let $\operatorname{Ind}\phi=-1$. Suppose that $\operatorname{Ind} R'\leq n-1$ is defined. Then $\operatorname{Ind} R\leq n$ if for any pair $F\subset G\ (\subset R)$ of a closed set $F$ and an open set $G$ there exists an open set $H$ with $F\subset H\subset G$ such that $\operatorname{Ind}(\bar{H}-H)\leq n-1$.

5) A covering which consists of two elements is called a binary covering.