45. A Note on Hausdorff Spaces with the Star-finite Property. II

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K. Morita [4] constructed, for every metric space R, a 0-dimensional metric space S and a closed continuous mapping f of S onto R such that $f^{-1}(x)$ is compact for every point x of R. The purpose of this note is to give an analogous proposition to this theorem for the case when R is paracompact Hausdorff. As for the terminologies and the notations used in this note, refer to my previous note [7].

Theorem 1. *Let f be a closed continuous mapping of a regular space R onto a topological space S with the star-finite property such that $f^{-1}(y)$ has the Lindelöf property for every point y of S. Then R has the star-finite property.*

Proof. Let $\mathfrak{U}$ be an arbitrary open covering of R. For every point y of S let $\mathfrak{U}_y=\{U_\alpha;\ \alpha\in A_y\}$ be a subcollection of $\mathfrak{U}$ which consists of countable elements such that $\mathfrak{U}_y$ covers $f^{-1}(y)$. Let $U_y=\smile\{U_\alpha;\ \alpha\in A_y\}$ and $V_y=S-f(R-U_y)$. Then V_y is an open neighborhood of y. Let $\mathfrak{B}=\{V_\beta;\ \beta\in B\}$ be a star-finite open covering of S which refines $\{V_y;\ y\in S\}$. Let us define a (single-valued) mapping φ of B into S such that $\varphi(\beta)=y$ yields $V_\beta\subset V_y$. Let $W_y=f^{-1}(V_y)$ and $W_\beta=f^{-1}(V_\beta)$. Then we can prove that $\mathfrak{W}=\{W_\beta\cap U_\alpha;\ \alpha\in A_{\varphi(\beta)},\ \beta\in B\}$ is a star-countable open covering of R.

To show that $\mathfrak{W}$ covers R, let x be an arbitrary point of R. Then there exists $\beta\in B$ such that $x\in W_\beta$. Since $V_\beta\subset V_{\varphi(\beta)}$, we get $W_\beta\subset W_{\varphi(\beta)}$. Since $W_{\varphi(\beta)}\subset U_{\varphi(\beta)}$ and $U_{\varphi(\beta)}=\smile\{U_\alpha;\ \alpha\in A_{\varphi(\beta)}\}$, there exists an $\alpha\in A_{\varphi(\beta)}$ such that $x\in U_\alpha$. Hence $\mathfrak{W}$ is an open covering of R. On the other hand the star-countability of $\mathfrak{W}$ is almost evident. Therefore we can conclude that R has the star-countable property. Since in general a regular space with the star-countable property has the star-finite property by Yu. Smirnov [9],[1] R has so and the theorem is proved.

Theorem 2. *Let R be a non-empty paracompact Hausdorff space. Then there exist a paracompact Hausdorff space A with* dim $A=0$ *and a closed continuous mapping f of A onto R such that $f^{-1}(x)$ is compact for every point x of R.*

Proof. Let $\{\mathfrak{F}_\lambda=\{F_\alpha;\ \alpha\in A_\lambda\};\ \lambda\in\Lambda\}$ be the collection of all locally finite colsed coverings of R. Let A be the aggregate of points a

1) This theorem is also almost essentially proved in Morita [5].