41. On Decomposition Theorems of the Vallée-Poussin Type in the Geometry of Parametric Curves

By Kanesiroo ISEKI

Department of Mathematics, Ochanomizu University, Tokyo

(Comm. by Z. SUETUNA, M.J.A., April 12, 1961)

1. **Introduction.** This is a continuation of our recent papers [1] to [3]. At the end of [2] we stated without proof a decomposition theorem for the measure-length induced by a locally rectifiable plane curve. It is the objective of the present note to prove this theorem in a slightly generalized form (see §4) and to obtain two further decomposition theorems concerning measure-length and spheric measure-length respectively. The last theorem will be applied elsewhere to derive a noteworthy property of the curvature of continuous parametric curves.

2. **Points of interjacence for a curve.** It is convenient to begin with two simple definitions. Let $\boldsymbol{R}^m$ be a Euclidean space of any dimension $m\geqq 2$ throughout the paper. Consider in $\boldsymbol{R}^m$ a parametric curve $\psi(t)$, defined and locally rectifiable on the real line $\boldsymbol{R}$. We shall term ψ *interjacent* at a point c of $\boldsymbol{R}$, if

$$|\psi(c-)-\psi(c+)|=|\psi(c-)-\psi(c)|+|\psi(c)-\psi(c+)|.$$

Further, a locally rectifiable, unit-spheric curve $\gamma(t)$ in $\boldsymbol{R}^m$ will be called *spherically interjacent* at c, if we have the angle-relation

$$\gamma(c-)\diamond\gamma(c+)=\gamma(c-)\diamond\gamma(c)+\gamma(c)\diamond\gamma(c+).$$

We may also call c *point of interjacence* of ψ and *point of spheric interjacence* of γ, in the respective cases.

The geometric meanings of the above two notions are easily seen. For instance, when $\psi(c-)\neq\psi(c+)$, the former notion means that the point $\psi(c)$ lies on the closed segment connecting the two points $\psi(c-)$ and $\psi(c+)$. (When the latter points coincide, interjacence of ψ at c is simply equivalent to its continuity at the same point.) We leave to the reader the consideration of the spheric case.

Evidently ψ [or γ] is interjacent [or spherically interjacent] wherever it is unilaterally (i.e. right-hand or left-hand) continuous.

3. **A lemma.** We shall now derive a result which includes the lemma left unproved in the final section of [3].

LEMMA. *Given ψ and γ as above, let S_* and Λ_* denote the measure-length induced by ψ and the spheric measure-length induced by γ, respectively. Then we have, for every point $t\in\boldsymbol{R}$,*

$$S_*(\{t\})=|\psi(t-)-\psi(t)|+|\psi(t)-\psi(t+)|,$$
$$\Lambda_*(\{t\})=\gamma(t-)\diamond\gamma(t)+\gamma(t)\diamond\gamma(t+).$$