106. "Foundation" and Formalism

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Has formalism really succeeded in laying the foundations of mathematics? If it has not, in what relation does it stand to foundation? Further, is the conception of foundation a legitimate one? These are the questions which I try to answer in this paper, for I am of the opinion that inquiry into them at the same time affords a clue to the ways of thinking underlying various views of the foundations of mathematics.

First, I take it that foundation is an operation aimed at examining whether an assertion which we make is correct and, if so, explaining why. As seen in this light, its basic attitude is not peculiar to it and mathematics, but is the one that constitutes the essential method of science in general. Only, science makes it its first duty to elucidate the unknown, while foundation is concerned to judge whether what is supposed to be known is really known. In other words, both of them have one and the same attitude in common, although they are interested in different spheres. Hence it may be concluded that the attitude in question is fairly sound.

Historically, too, in the evolution of the foundation of the differential and integral calculus which forms the prehistory of the foundations of mathematics, the method of ascertaining whether a seemingly self-evident matter was really an indisputable fact took the leading rôle, as with Cauchy, Dedekind and Cantor.

This method, however, cannot be taken for granted, because foundation presupposes the concept of the "true", which is thought to be meaningless. Speaking formularily, when the matter in hand is deduced from what has already been admitted to be true, it is regarded as founded. Now, the concept of the true involves in the last analysis the antiquated rule of "adquaetio rei et intellectus", which in turn postulates that the object has an existence independent from the subject, and that it can be grasped as a pure idea. This view belongs to realism in the epistemological sense and sanctions apriorism of some sort or other with regard to the truth or falsehood of a synthetic judgement. The way of thinking underlying Euclid's Elements is a typical specimen of it. But the possibility of grasping the object as an idea on which this view rests cannot be proved positively. In this respect, the phenomenological view of logical positivists is right, in so far as it holds that in real sciences,