131. On Generalized Laplace Transforms

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1. To investigate Laplace transforms of functionals is important relating two parts of analysis. On one hand it contributes to the developments of the theory of functional analysis and its applications, for instance, to the theory of partial defferential equations (Leray [1]). On the other hand it will contribute also to the investigations of the classical analysis, especially of the classical theory of Laplace transforms.

However, it seems to us that the systematic applications of functional analysis to the developments of the classical theory of Laplace transforms are still few at present.

Laplace transforms of distributions are investigated in detail by L. Schwartz [2]. However, he limited his considerations about Laplace transforms of the distribution T to the case such that $e^{-x\xi}T\in S'$. Such a limitation causes some confinements for the development and applications of the theory.

On the other hand, Fourier transforms of general distributions ($\in\mathfrak{D}'$) (not of tempered distributions $\in S'$) are investigated by E. M. Gelfand and G. E. Sylov [3], [4], [5] and L. Ehrenpreis [6]. These investigations, however, concern mainly to Fourier transforms, and the systematic theory of Laplace transforms is also not discussed.

In the preceding papers [7], [8], we considered divergent integrals $\int_0^\infty e^{i(\sigma+i\tau)s}v(\sigma,\tau,s)ds$ as functionals $\in\Phi'$. In case v is indepent of σ,τ, these integrals are Laplace integrals. Examples cited there are also of Laplace integrals. But the details of the theory were not discussed.

In this and the following papers we will consider the systematic theory of generalized Laplace integrals and its applications. In the preceding papers we used the dual of the space of the tensor product $Z(\sigma)\otimes D(\tau)$. But in this and following papers we will use mapping $\tau\to Z'(\sigma)$, though they are not essentially so different.

2. Let X^n, Y^n, Ξ^n, H^n be the n-dimensional real vector spaces, and Ξ^n+iH^n, X^n+iY^n be the n-dimensional complex vector spaces. We denote x, y, ξ, η, an element of X^n, Y^n, Ξ^n, H^n each, and call $\zeta=\xi+i\eta$, $z=x+iy$. To simplify notations, we use usually abbreviated writing as following: ηy means $\eta_1y_1+\cdots+\eta_ny_n$, $y\geqq 0$ means $y_1\geq 0,\cdots,$ $y_n\geq 0$ where $\eta=(\eta_1,\cdots,\eta_n)$ and $y=(y_1,\cdots y_n)$.