125. On Approximation and Uniform Approximation of Spaces

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All spaces under consideration are supposed to be completely regular. Concepts of approximation and uniform approximation of spaces are introduced. In Theorems 1–4 the difference between approximations and uniform approximations is shown. Finally, in Theorems 5 and 6 certain general results on approximations are stated.

Definition of approximations. Let $\boldsymbol{A}$ be a class of spaces. A space X is *approximated* by $\boldsymbol{A}$ in Y if $X\subset Y$ and for every x in $Y-X$ there exists an A in $\boldsymbol{A}$ with $X\subset A\subset Y-(x)$. The class of all spaces X which are approximated by $\boldsymbol{A}$ in $\beta(X)$ will be called the *closure* of $\boldsymbol{A}$ and will be denoted by cl $(\boldsymbol{A})$.

Definition of uniform approximations. Let $\boldsymbol{A}$ be a class of spaces. A space X is *uniformly approximated* by $\boldsymbol{A}$ in Y if $X\subset Y$ and for every closed (in Y) set $F\subset Y-X$ there exists an A in $\boldsymbol{A}$ with $X\subset A\subset Y-F$. The class of all spaces X which are uniformly approximated by $\boldsymbol{A}$ in $\beta(X)$ will be called the *uniform closure* of $\boldsymbol{A}$ and denoted by unif. cl $(\boldsymbol{A})$.

For convenience a class of spaces $\boldsymbol{A}$ will be called closed (uniformly closed) if cl $(\boldsymbol{A})=\boldsymbol{A}$ (unif. cl $(\boldsymbol{A})=\boldsymbol{A}$). From the definitions one can prove at once the following elementary formulae:

(1) $\boldsymbol{A}\subset$ unif. cl $(\boldsymbol{A})\subset$ cl $(\boldsymbol{A})$

(2) cl (cl $(\boldsymbol{A})$) = cl $(\boldsymbol{A})$

(3) unif. cl (unif. cl $(\boldsymbol{A})$) = unif. cl $(\boldsymbol{A})$

(4) unif. cl $(\boldsymbol{A}_1\cup\boldsymbol{A}_2)$ = unif. cl $(\boldsymbol{A}_1)\cup$ unif. cl $(\boldsymbol{A}_2)$.

Theorem 1. The uniform closure of the class $\boldsymbol{K}_\sigma$ of σ-compact spaces (countable unions of compact subspaces) is the class of all Lindelöf spaces (every open covering contains a countable subcovering).

Theorem 2. The closure of the class $\boldsymbol{K}_\sigma$ is the class of all Q-spaces (realcompact spaces in the terminology of Gillman and Jerison).

The proofs of both theorems are simple and may be left to the reader.

By a perfect mapping of X onto Y is meant a closed and continuous mapping of X onto Y such that the preimages of points are compact. One can prove the following results.

Theorems 1′ and 2′. The class $\boldsymbol{K}_\sigma$ in Theorems 1 and 2 may be replaced by each of the following classes: the class of all σ-compact locally compact spaces (=the class of all preimages under perfect