7. On Adjunction Spaces

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1. The Main Theorem. Let $\{C_\alpha \mid \alpha \in \Omega\}$ be a family of topological spaces. Let us consider a family of continuous maps $\{g_\alpha \mid \alpha \in \Omega\}$, where g_α is a continuous map defined on a *closed* subspace A_α of C_α into another topological space Y for each α. Then the *disjoint union* $W = Y \smile (\underset{\alpha\in\Omega}{\smile} C_\alpha)$ is a space with the topology defined as follows: a subset $V \subset W$ is open if and only if $V \frown Y$ is an open subset of Y and $V \frown C_\alpha$ is an open subset of C_α for each α. Now we define in W an equivalence relation as follows: Two points $x \in C_\alpha$ and $y \in Y$ are equivalent if and only if $g_\alpha(x) = y$; two points $x \in C_\alpha$ and $y \in C_\beta$ are equivalent if and only if $g_\alpha(x) = g_\beta(y)$; each point is equivalent to itself. We take Z to be the quotient space of W with respect to this equivalence relation and $p: W \to Z$ the natural projection; that is, a subset B of Z is open if and only if $p^{-1}(B)$ is an open subset in W. We call this space Z the *adjunction space obtained by adjoining* $\{C_\alpha\}$ *to* Y *by means of the continuous maps* $\{g_\alpha : A_\alpha \to Y\}$.

The adjunction space is one of the most important spaces in the homotopy theory. (Cf. Hu [1].) We shall consider here a set-theoretical property of this space. Namely we shall prove the following theorem.

Theorem 1. *Let* $\{C_\alpha \mid \alpha \in \Omega\}$ *be a family of topological spaces, and let* A_α *be a closed subspace of* C_α, g_α *a closed continuous map defined on* A_α *into another topological space* Y, *for each* $\alpha \in \Omega$. *Then each of the following properties for* Y *and all* C_α*'s, implies the same property for the adjunction space* Z, *obtained by adjoining* $\{C_\alpha\}$ *to* Y *by means of the continuous maps* $\{g_\alpha : A_\alpha \to Y\}$:

(1) *normality*, (2) *complete normality*,

(3) *perfect normality*, (4) *collectionwise normality*,

(5) $\mathfrak{m}$-*paracompactness and normality*,

where $\mathfrak{m}$ *is any infinite cardinal number.*

Here a topological space is called $\mathfrak{m}$-*paracompact* if any open covering of power $\leqq \mathfrak{m}$ admits a locally finite open refinement. This notion is due to K. Morita [3].

In his lecture on the obstruction theory of CW-complexes [4], G. W. Whitehead has introduced the notion of relative CW-complexes. (For the definition, see §3 below.) As an application of Theorem 1, we shall establish the following theorem.