5. *Rotationally Invariant Measures in the Dual Space of a Nuclear Space*

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The purpose of the present paper is to show that any rotationally invariant measure in a Hilbert space (more exactly, in the dual space of a nuclear space) is expressed as a superposition of Gaussian measures. The author intends to discuss this problem in details in another paper. So, we shall indicate the proof only briefly.

§ 1. **Preliminaries.** We shall explain our problem more exactly. Let L be a real topological vector space which is defined by countable Hilbertian norms and is nuclear. Let L^* be the dual space of L. For L and L^*, R. A. Minlos proved the following generalization of Bochner's theorem:

For every continuous and positive definite function $\chi(\xi)$ on L there exists a uniquely determined Borel measure μ on L^*, which fulfils the relation

$$\chi(\xi)=\int \exp [i(\xi, x)]d\mu(x). \tag{1}$$

Conversely, for any μ the relation (1) defines a continuous and positive definite function $\chi(\xi)$, the characteristic function of μ.

Now let H be the completion of L by a continuous Hilbertian norm $\|\cdot\|$. Then we may suppose $L\subset H\subset L^*$.

We shall call an orthogonal operator u on H a *rotation* of L, if it satisfies the following conditions:

1) u maps L onto L;

2) u is homeomorphic on L.

All the rotations of L form a group, which we shall call the *rotation group* of L and denote by $O(L)$. If we identify u and u^{-1*}, $O(L)$ can be regarded a transformation group of L^* onto itself.

Now let G be any group of homeomorphic transformations of L^* onto itself. From a given measure μ on L^*, we define the transformed measure $\tau_g\mu$ as follows:

$$\tau_g\mu(A)=\mu(gA), \text{ for any Borel set } A.$$

If $\mu=\tau_g\mu$ for any $g\in G$, then μ is called *G-invariant*. It $\tau_g\mu$ is absolutely continuous with respect to μ for any $g\in G$, then μ is called *G-quasi-invariant*. Finally, μ is called *G-ergodic*, if μ is G-quasi-invariant and the condition $A=gA$ (for all $g\in G$) implies $A=\phi$ or $A=L^*$ modulo nullsets. In the case of $G=O(L)$, we simply call μ O-invariant or O-ergodic.