4. Decomposition of Representations of the Three-Dimensional Lorentz Group

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The purpose of the present paper is the explicit description of decomposition of the unitary representations of the three-dimensional Lorentz group, which are constructed on factor spaces, into irreducible representations. We solve this problem by infinitesimal method. This problem arises as one step of decomposing tensor products of irreducible representations of the inhomogeneous Lorentz group into irreducible ones. The proof of the results of the present paper and the details about the last problem will be published in other papers.

§ 1. In order to include the so-called spinor representations, we consider the real special linear group G of second order. G is the two-fold covering group of the three-dimensional Lorentz group. Now we shall give some definitions which are necessary to describe our problem and results exactly.

G is generated by three subgroups of the following types:

$$S=\left\{s(\theta)=\begin{pmatrix}\cos(\theta/2), & -\sin(\theta/2)\\ \sin(\theta/2), & \cos(\theta/2)\end{pmatrix};\ -2\pi<\theta\leq 2\pi\right\},$$

$$D=\left\{d^{\pm}(\zeta)=\pm\begin{pmatrix}\exp(\zeta/2), & 0\\ 0, & \exp(\zeta/2)\end{pmatrix};\ -\infty<\zeta<\infty\right\},$$

$$L=\left\{l^{\pm}(t)=\pm\begin{pmatrix}\mathrm{ch}(t/2), & \mathrm{sh}(t/2)\\ \mathrm{sh}(t/2), & \mathrm{ch}(t/2)\end{pmatrix};\ -\infty<t<\infty\right\}.$$

For arbitrary unitary representation $\mathfrak{R}$ of G, denote the corresponding operators to the generators of Lie algebra with respect to these parameters by $H(S,\mathfrak{R})$, $H(D,\mathfrak{R})$, $H(L,\mathfrak{R})$ respectively.

The factor space $S\backslash G$ can be imbedded in G by the correspondence of coset to its representative e or $d^{+}(\zeta)s(\theta)$ $(0<\zeta<\infty,\ -\pi<\theta\leq\pi)$. In the same way, for the case of $D\backslash G$, the element e or $l^{+}(t)s(\theta)$ $(-\infty<t<\infty,\ -\pi<\theta\leq\pi)$ are representatives.

§ 2. The representations to be decomposed are the so-called induced representations of G from one-dimensional representations of S or D, that is, $\exp(ik\theta)$ (k: integer or half-integer) of S, and $\exp(i\tau\zeta)$ (τ; real) of D, or $\pm\exp(i\tau\zeta)$ for spinor case. The spaces of these representations are L^2_μ by the G-invariant measure μ over the factor space $\Omega=S\backslash G$ or $D\backslash G$ respectively, and the operator U_g of representation is defined by a function $\alpha(\omega, g)$ over $\Omega\times G$ for any element $f(\omega)$ of L^2_μ: $(U_g f)(\omega)\equiv\alpha(\omega, g)f(\omega\cdot g)$. The multiplier $\alpha(\omega, g)$ is given as follows: