25. *Representations of Compact Groups Realized by Spherical Functions on Symmetric Spaces*

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1. The problem of determining the irreducible representations of a connected compact semisimple Lie group G realized by spherical functions on a symmetric Riemannian space G/K was first treated by E. Cartan [1]. In the present note we shall give a more explicit determination of these representations by means of "Satake diagrams". Our theory could be founded on the basis of the fundamental result of Cartan ([1], p. 241). It should be noticed however that, although this result is valid, its proof in [1] was not complete. So we shall start anew from the beginning. The detailed discussion with proofs will appear elsewhere.

2. Let G be a compact group and K be a closed subgroup of G. The totality $C(G/K)$ of complex valued continuous functions on G/K becomes the representation space of the representation $(T, C(G/K))$ of G if we define $T_g f = f \circ g^{-1}$, $f \in C(G/K)$. An element of an irreducible invariant subspace of $C(G/K)$ under this representation is called a spherical function on G/K. A representation (ρ, V) of G is called a representation realized by spherical functions on G/K if (ρ, V) is equivalent to one which is an irreducible component of $(T, C(G/K))$. It is easily seen that an irreducible representation (ρ, V) of G is realized by spherical functions on G/K if and only if $\rho(K)$ has a non-zero invariant in V (cf. E. Cartan [1]).

The most interesting case, to which we confine ourselves, is when G is a Lie group and G/K is a symmetric Riemannian space. In this case spherical functions are characterized as the simultaneous eigenfunctions of all invariant linear differential operators on G/K (cf. M. Sugiura [3]).

3. Let σ be an involutive automorphism of a connected compact semisimple Lie group G and K be the totality of fixed points under σ. K is a closed subgroup of G and the coset space G/K has the structure of a symmetric Riemannian space. Let $\mathfrak{g}$ and $\mathfrak{k}$ be the Lie algebras of G and K respectively. For any subspace V of $\mathfrak{g}$, we denote by $V^{\perp}$ the orthogonal complement of V with respect to the Killing form $(X, Y) = Tr\ adXadY$ which is negative definite on $\mathfrak{g}$.

Put $\mathfrak{p} = \mathfrak{k}^{\perp}$. Let $\mathfrak{a}$ be a maximal abelian subalgebra in $\mathfrak{p}$, and $\mathfrak{h}$ be a maximal abelian subalgebra in $\mathfrak{g}$ containing $\mathfrak{a}$. $\mathfrak{h}$ is a Cartan