20. A Remark on Mapping Spaces

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(Comm. by K. KUNUGI, M.J.A., March 12, 1962)

1. Let X and Y be topological spaces. We shall consider the set of all continuous mappings of X into Y. This set is turned into a topological space by the compact-open topology; this topology is defined by selecting as a sub-basis for the open sets the family of sets $T(K, G)$, where K ranges over all the compact subsets of X and G ranges over all the open subsets of Y and $T(K, G)$ denotes the set of all continuous mappings f of X into Y such that $f(K)\subset G$. As usual, we write Y^X for the *mapping space* thus obtained.

Now let X and Y be Hausdorff spaces, and let Z be a topological space. Then, with any continuous mapping f of $X\times Y$ into Z, there is associated a mapping f^* from Y to the mapping space Z^X by the formula

$$[f^*(y)](x)=f(x, y).$$

The correspondence $f\to f^*$ defines a one-to-one mapping

$$\theta: Z^{X\times Y}\to(Z^X)^Y.$$

K. Morita has shown in [2] that this mapping θ is always a homeomorphism into. But it is not necessarily a homeomorphism onto. We shall consider Hausdorff spaces, for which the mapping θ is a homeomorphism onto for any locally compact Hausdorff space Y and any topological space Z, and denote the class of such spaces by Θ.

In [2], the following notion has been introduced. A Hausdorff space X will be said to *have the weak topology with respect to compact sets in the wider sense* if a subset A of X such that $A\frown K$ is closed for every compact subset K of X is necessarily closed. We shall denote by $\mathfrak{B}$ the class of Hausdorff spaces having the weak topology with respect to compact sets in the wider sense. It is known that locally compact spaces and CW-complexes in the sense of J.H.C. Whitehead belong to the class $\mathfrak{B}$. Moreover, it has been shown in [2, Theorem 4] that $\mathfrak{B}\subset\Theta$, but it is unknown to the author whether $\mathfrak{B}=\Theta$ or not.

Now, let X be a topological space and $\{A_\alpha\}$ a closed covering of X. The space X is said to *have the weak topology with respect to* $\{A_\alpha\}$ *in the wider sense* if any subset of X whose intersection with each A_α is closed is necessarily closed. Then, the main result of this note is stated as follows.

Theorem 1. *Let X be a Hausdorff space having the weak*