59. On (m, n)-Mutant in Semigroup

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In his paper [2], A. A. Mullin introduced a new concept *mutant* concerned with biological computation (in particular, with physico-logical view) by Professor Heinz von Foester. For detail on biological computer and related subjects on relay switching field, see Engineering Outlook, the University of Illinois, vol. 1, no. 8 (1960) and A. A. Mullin [1].

Let S be a semigroup. A subset A in S is called *(m, n)-mutant* in S if $A^m \subset S - A^n$. The $(2, 1)$-mutant is a mutant in the sense of Mullin (2).

Proposition 1. *Every subset of a (m, n)-mutant of S is a (m, n)-mutant of S.*

Proof. Let B be a subset of the (m, n)-mutant A of S, then $B \subset A$ implies $B^k \subset A^k$ for every k. Hence

$$B^m \subset A^m \subset S - A^n \subset S - B^n.$$

Proposition 2. *Let $A_\alpha (\alpha \in \Lambda)$ be (m, n)-mutants of S, then $\bigcap_{\alpha \in \Lambda} A_\alpha$ is a (m, n)-mutant of S, where $\bigcap_{\alpha \in \Lambda} A_\alpha$ is non-empty.*

Proof. From Proposition 1. Let φ be a homomorphism from S_1 into S_2, then $\varphi(a) \cdot \varphi(b) = \varphi(ab)$ for every $a, b \in S$.

Proposition 3. *Let A be a (m, n)-mutant of S_1. If $\varphi(S_1 - A^n) \subset S_2 - \varphi(A^n)$, then $\varphi(A)$ is a (m, n)-mutant in S_2.*

Proof. Proposition follows from

$$(\varphi(A))^m = \varphi(A^m) \subset \varphi(S_1 - A^n) \subset S_2 - \varphi(A^n) = S_2 - (\varphi(A))^n.$$

Proposition 4. *The inverse image under a homomorphism φ of a (m, n)-mutant is a (m, n)-mutant.*

Proof. Let homomorphism φ be $\varphi\colon S_1 \to S_2$, and suppose that B is a (m, n)-mutant of S_2. Let a $\in \varphi^{-1}(B)$, then we can find an element b in B such that $b = \varphi(a)$. Then $\varphi(a^m) = (\varphi(a))^m = b^m \in S_2 - B^n$. Hence $a^m \in \varphi^{-1}(S_2 - B^n) = S_1 - \varphi^{-1}(B^n) = S_1 - (\varphi^{-1}(B))^n$. This shows that $(\varphi^{-1}(B))^n \subset S_1 - (\varphi^{-1}(B))^n$.

A subset A of S is called to be a maximal (m, n)-mutant, if there is no (m, n)-mutant of S containing A.

By Zorn's lemma, we have

Proposition 5. *Every (m, n)-mutant is contained in a maximal (m, n)-mutant.*

We shall give an interesting example on (m, n)-mutant.