

75. On Distributions and Spaces of Sequences. I

General Theory

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1. P. Dirac [1] introduced improper function δ when he constructed his theory of representation in quantum mechanics. Further L. Schwartz [2] gave a mathematical foundation of this function using his theory of distributions.

Improper functions in quantum mechanics whose singularities are like δ , thus, can be treated in a mathematically rigorous way.

In quantum field theory, however, not the single δ function but the multiplications of improper functions of same sort of singularity appear [4] and it is widely recognized that the difficulties of the present quantum field theory originate from these very multiplications.

The multiplication of δ type functions are considered by some authors [5] [6] [7] [8]. H. König constructed algebraically the multiplication using tensor product, but he dared not extend his consideration to deeper investigation of the relation between the extended multiplication and the ordinary multiplication defined by L. Schwartz.

E. Stueckelberg and A. Petermann considered the multiplication accompanied with division and carry out his calculation using these products. W. Güttinger defined the multiplication by the same sort of calculation, but considered it by the way of extension of the research of König. Both of these definitions are not perfectly rigorous. In addition to these defects, all of three definitions lack in associativity and commutativity, which is very inconvenient for the application in field theory. For example, we can not attain always unique results in calculation of the vertex function [12].

Since the multiplication of two arbitrary distributions is not always a distribution we shall need to extend the notion of distribution in order to study the multiplication of general non-function distributions.

In this and the following papers we extend the notion of distribution to the spaces of sequences and introduce the generalized distribution and consider various operations including the multiplication in these spaces, thus try to construct the new formalism for calculation of some sorts of singular improper functions.