72. On (m, n)-Antiideals in Semigroup

By Kiyoshi ISÉKI
Kobe University

(Comm. by K. KUNUGI, M.J.A., July 12, 1962)

In my Note [2], we considered properties of mutants in semigroup. The concept of the (m, n)-mutant is a generalization of the concept of the mutant by A. A. Mullin. Recently N. Chomsky, G. A. Miller and Y. Bar-Hillel and his colleagues have shown the usefulness of theory of semigroups for linguistics. In his paper [4], S. Schwarz defined antiideals and has shown to be useful for the study of structures in semigroup. On the other hand, S. Lajos introduced an interesting concept: (m, n)-ideals, which is a generalization of ideals in semigroup. S. Lajos ([1], [2]) has proved some important properties on (m, n)-ideals in semigroup.

In this note, we shall introduce the concept of (m, n)-antiideals in semigroups. This concept is very similar with mutants in semigroup.

Definition. A subset A of a semigroup S is called a *left* (m, n)-*antiideal* of S if $SA^m \cap A^n = \phi$. A subset A of S is called a *right* (m, n)-*antiideal* of S if $A^m S \cap A^n = \phi$.

Any $(1, 1)$-antiideal is an antiideal in the sense of S. Schwarz [4]. *If a semigroup S has a left unit, then there is no left (m, m)-antiideal in S.*

To prove it, left e be a left unit of S. Then $ea=a$ implies $SA \supset A$. Hence we have $SA^m \supset A^m$, and therefore $SA^m \cap A^m \neq \phi$. This shows that there is no left (m, m)-antiideal in S having a left unit.

We shall prove the following proposition, which shows essentially difference from the concept of (m, n)-mutant, and an interesting fact in the view of semigroup for mathematical linguistics.

Theorem. There are no left (right) (m, n)-antiideals $(m<n)$ in any semigroups.

Proof. For any set A of S, we have

$$SA \supset A^2.$$

Hence, $SA^m \supset A^{m+1}$, and this shows $SA^m \cap A^{m+1} \neq \phi$.

Let n be greater than m, then we have the following sequence:

$$SA^m \supset S^{n-m} A^m \supset \cdots \supset SA^{n-1} \supset A^n.$$

This shows $SA^m \cap A^n \neq \phi$. Therefore there is no left (m, n) antiideal in any semigroup.

On the other hand there is a semigroup having left (right) (m, n)-antiideals $(m \geq n)$. Consider the additive semigroup S of