70. *A Remark on the Concept of Channels*

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1. In the mathematical theory of information (cf. [4]), a message is represented by a sequence of alphabets. Considerable messages are represented by the points of the infinite product space of alphabets which is called the message space associated with a measure on the field of all measurable subsets generated by the cylinder sets. A *channel* $[X, \nu, Y]$ is represented by a function $\nu(x, E)$ where x runs through an input message space X and E varies all measurable subsets of an output message space Y, which is usually assumed to satisfy the following two conditions:

(i) For almost all x, $\nu(x, E)$ is a probability measure on Y, and

(ii) for all E, $\nu(x, E)$ is a measurable function of x.

If $[X, \nu, Y]$ is a channel and if μ is a probability measure on X, then

$$\lambda(E)=\int_X \nu(x, E)d\mu(x) \tag{1}$$

defines a probability measure λ on Y. Being defined by

$$\lambda=K_*\mu, \tag{2}$$

K_* maps the space of all probability measures on X into that of Y.

In the above description, it is not essential, as recently observed by H. Umegaki, that the measure space X is the direct product of alphabets. He discussed the case that the output and input spaces are simply measure or measurable spaces. Again, in his case, (2) defines a linear mapping K_* which carries the probability measures into the probability measures.

Being considered probability measures as normal states of the abelian von Neumann algebras of all bounded measurable functions on the output and input spaces, Umegaki's discussion suggests us to generalize the concept of channels for not necessarily commutative von Neumann algebras which will be given a preliminary discussion in the below.

2. Basing on the definitions of Dixmier [2], let us suppose that A_* and B_* are the subconjugate space of all ultraweakly continuous linear functionals on von Neumann algebras A and B respectively. A positive linear transformation K_* defined on A_* with its range in B_* is called a *generalized channel* provided that K_* preserves the norm of positive elements:

$$\|K_*\rho\|=\|\rho\|, \tag{3}$$