67. On a Ya. B. Rutickii's Theorem Concerning a Property of the Orlicz Norm

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Let $\boldsymbol{R}$ be a modulared semi-ordered linear space[1] and its modular be $m(x)(x \in \boldsymbol{R})$, and suppose the semi-regularity[2] of $\boldsymbol{R}$.

Concerning the property of the Orlicz norm:

$$(1) \qquad \lim_{\|u\|_M \to \infty} \frac{1}{\|u\|_M} \int_G M[|u(t)|]dt = \infty,$$

where G is a bounded closed set in finite-dimentional Euclidian space and M is a N-function (see [2]), Ya. B. Rutickii [4] gave the following theorem.

Theorem 1. *In order that* (1) *be fulfilled, it is necessary and sufficient that there exists a function* $f(u)$ $(0 \leqq u < \infty)$, *satisfying the condition*

$$(2) \qquad \lim_{u \to \infty} f(u) = \infty$$

and such that for every v *and all sufficiently large values of* u *the inequality*

$$(3) \qquad M(uv) \geqq uf(u)M(v)$$

be fulfilled.

The Orlicz space $\boldsymbol{L}_M^*$ is a modulared space on which the modular is defined as

$$(4) \qquad m(x) = \int_G M[|x(t)|]dt \qquad (x \in \boldsymbol{L}_M^*).$$

Then, (1) is written as

$$(5) \qquad \lim_{\|x\| \to \infty} m(x)/\|x\| = \infty$$

where $$\|x\| = \inf_{\xi > 0} \frac{1 + m(\xi x)}{\xi} \; (= \|x\|_M)^{3)}$$

The purpose of this paper is to prove the following theorem.

Theorem 2. *Let* $\boldsymbol{R}^m$ *be a modulared semi-ordered linear space. Then, in order that* (5) *be fulfilled, it is necessary and sufficient*

1) Namely, $\boldsymbol{R}$ is a conditionally vector lattice, in the sense of G. Birkhoff, on which a functional $m(x)$ is defined, and then such space is denoted by $\boldsymbol{R}^m$. (see [3, §35]).

2) $\boldsymbol{R}$ is said to be *semi-regular*, if for any $o \neq x \in \boldsymbol{R}$ there exists an element $\bar{x} \in \overline{\boldsymbol{R}}$ such that $\bar{x}(x) \neq 0$, where $\overline{\boldsymbol{R}}$ is the totality of all linear functionals $\bar{x}$ satisfying that $x_\lambda \downarrow_{\lambda \in \Lambda} 0$ implies $\inf_{\lambda \in \Lambda} |\bar{x}(x_\lambda)| = 0$.

3) See Theorem 10.5 in [2].