163. *Hilbert Transforms in the Stepanoff Space*

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1. Introduction. By the Stepanoff space we mean the set of measurable function such that for some positive number $l>0$ there exists a constant K and we have

$$\sup_{-\infty<x<\infty} \frac{1}{l}\int_x^{x+l} |f(t)|^p dt \leqq K \quad (1\leqq p\leqq \infty). \tag{1.1}$$

We denote these classes by S^p. This norm is firstly introduced by W. Stepanoff [4] for the study of almost periodic functions.

The purpose of this paper is to find under what condition does the Hilbert transform of a function of the class S^2 belong to the same class again?

The Hilbert transform is defined by the following formula

$$\tilde{f}(x)=1/\pi\int_{-\infty}^{\infty} f(t)/(x-t)dt. \tag{1.2}$$

We understand this singular integral as the Cauchy sense. It does not always define and we assume its existence for almost all x.

One of the important property of Hilbert transform is that it commutes with translations and dilatations. These are

$$F(t)=f(t+a) \quad \text{implies} \quad (\tilde{F})(t)=(\tilde{f})(t+a). \tag{1.3}$$

$$F(t)=f(\lambda t) \quad \text{implies} \quad (\tilde{F})(t)=(\tilde{f})(\lambda t). \tag{1.4}$$

These properties are pointed out explicitely by M. Cotlar [1]. The author have learned this through mimeorgraphed papers presented by Dr. Y. M. Chen of the Hong-Kong University. The author thanks him for his kind considerations.

2. Equivalence between two norms. We consider the second norm. For all $T\geqq 1$ and all real number x, $(1/2T)\int_{-T}^{T} |f(t+x)|^p dt$ is uniformly bounded. That is, there exists a constant K such as

$$\frac{1}{2T}\int_{-T}^{T} |f(t+x)|^p dt \leqq K' \quad (-\infty<x<\infty,\ T\geqq 1).$$

And thus we get

$$\overline{\lim_{T\to\infty}}\frac{1}{2T}\int_{-T}^{T} |f(x+t)|^p dt \leqq K' \quad \text{unif. } x. \tag{2.1}$$

We denote this uniform norm as (2.1).

Lemma 1. The two norms (1.1) and (2.1) are equivalent.

If (1.1) consists for some $l>0$ then it does also for any other $l'>0$