9. On Summability $[c, k]$ and Summability $[R, k]$ of Laplace Series

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1. If $f(\theta, \phi)$ be a function defined for the range $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$, the Laplace series associated with this function on the sphere S is

$$f(\theta, \phi) \sim \frac{1}{2\pi} \sum_{n=0}^{\infty} (n+1/2) \iint_S f(\theta', \phi') P_n(\cos \gamma) \sin \theta' d\theta' d\phi', \tag{1.1}$$

where

$$\cos \gamma = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\phi - \phi'),$$

and $P_n(x)$ denotes the n-th Legendre polynomial.

The generalized mean value of $f(\theta, \phi)$ is given by

$$f(\gamma) = \frac{1}{2\pi \sin \gamma} \int_{C\gamma} f(\theta', \phi') \, dS', \tag{1.2}$$

where the integral is taken along the small circle whose centre is (θ, ϕ) on the sphere and whose curvilinear radius is γ.

The series

$$\sum_{n=0}^{\infty} u_n \tag{1.3}$$

is said to be strongly summable (c, k) or summable $[c, k]$ to the sum S, if

$$\sum_{\nu=0}^{n} | s_\nu^{(k-1)} - s | = 0(n), \tag{1.4}$$

where $s_\nu^{(k-1)}$ denotes the ν-th cesaro mean of order $(k-1)$ of the series (1.3).

Again, we say that the series (1.3) is strongly summable (R, k) or summable $[R, k]$ to the sum S, if

$$\sum_{\nu=0}^{n} \frac{| s_\nu^{(k-1)} - s |}{\nu+1} = 0 \, (\log n). \tag{1.5}$$

The object of this paper is to obtain some new results for the series (1.1) on its $[c, k]$ and $[R, k]$ summability.

We prove the following theorems:

Theorem 1: *If*

$$\varphi(t) \equiv \int_t^{\delta} \frac{|\phi(\gamma)|}{\gamma} d\gamma = 0\left[t \left(\log \frac{1}{t} \right)^{\alpha} \right], \quad (\alpha > 0) \tag{1.6}$$

as $t \to 0$, $(0 < \delta \leq \pi)$, then

$$\sum_{\nu=0}^{n} | \sigma_\nu^{(k)}(\gamma) - \sigma | = 0[n(\log n)^{\alpha}], \quad (0 \leq k \leq 1), \tag{1.7}$$