50. On the Maximum Principle for Quasi-linear Parabolic Equations of the Second Order

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Introduction. In this note we shall discuss the maximum-minimum property of solutions of general quasi-linear parabolic equations of the second order. For linear parabolic equations such property, known as the maximum principle, has been exhaustively exploited and has been playing an essential part in the study of both linear and non-linear parabolic equations. As is well known, the strongest results in this connection have been given by Nirenberg [4]. It seems, however, that the maximum-minimum property for quasi-linear parabolic equations has hitherto been investigated unsatisfactorily and that the deeper investigation might enable us to establish results of more or less use.

The main purpose of this note is to give an extension of the so-called "strong maximum principle" established by Nirenberg [4] to the case of quasi-linear parabolic equations. Section 2 is devoted to this extension. We note here that this is an analogue of the maximum principle proved by the author [5]. In section 1 we formulate without proofs a very simple maximum principle and some of its consequences. In both sections from the maximum principles immediately follow the uniqueness theorems for the first boundary value problem and the Harnack type convergence theorems.

Let D denote a bounded domain in the $(n+1)$-dimensional (x, t)-space, bounded by two hyperplanes $t=0$ and $t=T>0$, and by a surface S lying between these hyperplanes. $\overline{D}$ denotes the closure of D, B the lower basis of $D: B=\overline{D}\cap\{t=0\}$, and ∂D the normal boundary of D consisting of S and B.

Quasi-linear parabolic equations we are concerned with are of the type

$$(1)\qquad \sum_{i,j=1}^{n} a_{ij}(x, t, u, \operatorname{grad} u)\frac{\partial^2 u}{\partial x_i \partial x_j}-\frac{\partial u}{\partial t}=f(x, t, u, \operatorname{grad} u)$$

$$(x=(x_1,\cdots,x_n),\ \operatorname{grad} u=(\partial u/\partial x_1,\cdots,\partial u/\partial x_n)).$$

The functions $a_{ij}(x, t, u, p)$ and $f(x, t, u, p)$ are defined in the domain $\mathfrak{D}$: $\{(x, t)\in D, |u|<\infty, ||p||<\infty\}$ and are bounded in any compact subset of $\mathfrak{D}$. By a solution of the first boundary value problem for the equation (1) we mean a function $u(x, t)$ which is continuous in $\overline{D}$, bounded