# 94. Notes on $(m, n)$-Ideals. I

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Let $S$ be a semigroup. A subsemigroup $A$ of $S$ is called $(m, n)$-ideal of $S$, if $A$ satisfies the condition

$$A^m S A^n \subseteq A \tag{1}$$

where $m$, $n$ are non-negative integers ($A^m$ is suppressed if $m=0$). By a proper $(m, n)$-ideal we mean an $(m, n)$-ideal, which is a proper subset of $S$. The concept of $(m, n)$-ideal is a generalization of one-sided (left or right) ideals in semigroups and was introduced in [2]. (See also [3], [4], [5], [6] and [1].)

In this note we prove some theorems on $(m, n)$-ideals.

**Theorem 1.** *Let $S$ be a semigroup, $T$ be a subsemigroup of $S$ and let $A$ be an $(m, n)$-ideal of $S$. Then the intersection $A \cap T$ is an $(m, n)$-ideal of the semigroup $T$.*

*Proof.* The intersection $A \cap T$ evidently is a subsemigroup of $S$. We show that $A \cap T$ satisfies (1). First, we see that

$$(A \cap T)^m T (A \cap T)^n \subseteq A^m S A^n \subseteq A \tag{2}$$

because of $A$ is an $(m, n)$-ideal of $S$. Secondly

$$(A \cap T)^m T (A \cap T)^n \subseteq T^m T T^n \subseteq T \tag{3}$$

therefore (2) and (3) imply

$$(A \cap T)^m T (A \cap T)^n \subseteq A \cap T,$$

that is the intersection $A \cap T$ is an $(m, n)$-ideal of $T$.

**Theorem 2.** *Let $S$ be a semigroup, $A$ be an $(m, n)$-ideal of $S$ and let $B$ be a subset of $S$ satisfying either $AB \subseteq A$ or $BA \subseteq A$. Then the products $AB$ and $BA$ are $(m, n)$-ideals of $S$ ($m, n$ are positive integers).*

*Proof.* Suppose that e.g. the condition $AB \subseteq A$ is fulfilled. Hence

$$(AB)(AB) \subseteq A \cdot AB \subseteq AB,$$

i.e. $AB$ is a subsemigroup of $S$. On the other hand

$$(AB)^m S (AB)^n \subseteq A^m S A^{n-1} \cdot (AB) \subseteq AB$$

because of $A$ is an $(m, n)$-ideal of $S$. Thus $AB$ is an $(m, n)$-ideal of $S$.

We prove that $BA$ is also $(m, n)$-ideal of $S$. Since

$$(BA)(BA) = B(AB)A \subseteq BA \cdot A \subseteq BA,$$

$BA$ is a subsemigroup of $S$. From the condition $AB \subseteq A$ it follows, that

$$(BA)^m S (BA)^n \subseteq B \cdot A^m S A^n \subseteq BA,$$

therefore $BA$ is also an $(m, n)$-ideal of $S$.

Analogously we can prove our theorem if the condition $BA \subseteq A$