93. *Note on Balayage and Maximum Principles*

By Masanori KISHI

Mathematical Institute, Nagoya University

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1. Let Ω be a locally compact Hausdorff space, every compact subset of which is separable, and G be a positive lower semicontinuous kernel on Ω such that $G(x, y)$ is locally bounded at any point $(x, y) \in \Omega\times\Omega$ with $x \neq y$. The *adjoint* kernel $\check{G}$ of G is defined by $\check{G}(x, y) = G(y, x)$. Given a positive measure μ, its potential $G\mu(x)$ and adjoint potential $\check{G}\mu(x)$ are defined by

$$G\mu(x)=\int G(x, y)d\mu(y) \quad \text{and} \quad \check{G}\mu(x)=\int \check{G}(x, y)d\mu(y)$$

respectively.

This note is a summary of some relations among the balayage principle and related maximum principles in the potential theory. The details will be published later elsewhere.

2. **Definitions.** We say that a property holds G-*p.p.p.* on a subset $X\subset\Omega$, when any compact subset of the set of points in X at which the property is missing does not support any positive measure $\nu\neq 0$ with finite G-energy $\int G\nu d\nu$.

(I) *Continuity principle.* For any positive measure μ with compact support $S\mu$, if the restriction of $G\mu(x)$ to $S\mu$ is finite and continuous, then $G\mu(x)$ is finite and continuous in the whole space Ω.

(II) *Balayage principle.* For any compact set K and any positive measure μ, there exists a positive measure μ', supported by K, such that

$$G\mu'(x)\leq G\mu(x) \quad \text{in } \Omega,$$
$$G\mu'(x)=G\mu(x) \quad G\text{-}p.p.p. \text{ on } K.$$

(III) *Equilibrium principle.* For any compact set K, there exists a positive measure μ, supported by K, such that

$$G\mu(x)\leq 1 \quad \text{in } \Omega,$$
$$G\mu(x)=1 \quad G\text{-}p.p.p. \text{ on } K.$$

(IV) *Domination principle.* For a positive measure μ with compact support and finite G-energy and for a positive measure ν with compact support, an inequality $G\mu(x)\leq G\nu(x)$ on $S\mu$, the support of μ, implies the same inequality in Ω.

(V) *Maximum principle.* For a positive measure μ with compact support, the validity of an inequality $G\mu(x)\leq 1$ on $S\mu$ implies that of the same inequality in Ω.