44. On the Theorems of Constantinescu-Cornea

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1. Let f be a non-constant analytic mapping from a hyperbolic Riemann surface R into an arbitrary Riemann surface R'. C. Constantinescu and A. Cornea defined[1] a cluster set and developed the theorem of Riesz and the theorem of Fatou. Their cluster set is defined by means of the operator I and the argument is carried out mechanically. We shall give here an intuitive interpretation of this cluster set by the notion of thinness due to L. Naïm.[2]

2. We can define the Martin boundary Δ of R, and the set of minimal boundary points Δ_1.[3] For $s\in\Delta_1$ and an open subset G in R Constantinescu-Cornea defined

$$\underset{G}{I}K_s=\sup\{u(p)\,;\ u\in HP(\eta),\ u\leq K_s \text{ in } G\},$$

where K_s is the minimal positive harmonic function in R corresponding to s and η is an identity mapping from G into R. By definition, $u\in HP(\eta)$ if and only if for every relatively compact open set $G_1\subset R$, $H_u^{G\cap G_1}=u$ in $G\cap G_1$ where $H_u^{G\cap G_1}$ denotes the solution of the Dirichlet problem with the boundary function u on $\partial(G\cap G_1)\cap G$[4] and 0 else where. Further, if $\underset{G}{I}K_s\not\equiv 0$ they set $s\in\Delta_1(G)$ and the cluster set is defined as follows:

$$\hat{M}_f(s)=\bigcap_{s\in\Delta_1(G)}\overline{f(G)},$$

$\overline{f(G)}$ is the closure of $f(G)$ in $\hat{R}'$ (compactification of R').[5]

We shall here remark that the set $\Delta_1(G)$ permits the potential theoretic view. In fact, Constantinescu-Cornea showed that the following equality holds in G:

$$\underset{G}{I}K_s=K_s-H^G_{K_s}.\text{[6]}$$

1) C. Constantinescu and A. Cornea: Über das Verhalten der analytischen Abbildungen Riemannscher Flächen auf dem idealen Rand von Martin. Nagoya Math. J., **17**, 1-87 (1960).

2) L. Naïm: Sur le rôle de la frontière de R. S. Martin dans la théorie du potentiel. Ann. Inst. Fourier, **7**, 183-281 (1957).

3) For the construction and the properties of the Martin boundary see L. Naïm, l.c., also M. Parreau: Sur les moyennes des fonctions harmoniques et analytiques et la classification des surfaces de Riemann. Ann. Inst. Fourier, **3**, 103-197 (1952). R. S. Martin: Minimal positive harmonic functions. Trans. Amer. Math. Soc., **49**, 137-172 (1941).

4) $\partial(G\cap G_1)$ denotes the boundary of $G\cap G_1$.

5) Cf. Constantinescu-Cornea, l.c., S. 44.

6) Cf. Constantinescu-Cornea, l.c., Hilfssatz 4, S. 21.