33. *On the Representation of Large Even Integers as Sums of a Prime and an Almost Prime*

By Miyoko UCHIYAMA*) and Saburô UCHIYAMA**)

(Comm. by Zyoiti SUETUNA, M.J.A., March 12, 1964)

As is well known, the classical Goldbach problem, which remains still unsolved, is to prove that every even integer $\geqq 6$ is a sum of two odd prime numbers. In 1948 A. Rényi [3] proved that every even integer $\geqq 6$ is representable as a sum of a prime and an almost prime, i.e. an integer >1 with a bounded number of prime factors. Recently Ch.-D. Pan [2] sharpened this result by showing that every large even integer is a sum of a prime and an almost prime with at most five prime factors. Our aim in the present note is to prove the following theorem, which constitutes an improvement of this result of Pan:

Theorem. *Every sufficiently large even integer can be represented as a sum of a prime and an almost prime which is composed of at most four prime factors.*

Our proof of this theorem runs substantially on the same lines as in Pan [2].

By the same method we can also prove the following result: for every fixed integral value of $k\geqq 1$ there exist infinitely many primes p with $V(p+2k)\leqq 4$, where $V(m)$ denotes the total number of prime factors of m.

It is of some interest to note that Y. Wang [5] has proved under the extended Riemann hypotheses for Dirichlet L-functions that every sufficiently large even integer is representable as a sum of a prime and an almost prime with at most three prime factors and that for every fixed integral value of k there are infinitely many primes p with $V(p+2k)\leqq 3$.

1. We begin with reproducing the Fundamental Theorem of Pan [2] in a slightly refined form.

Let k and l be two integers such that $k\geqq 1$, $0\leqq l\leqq k-1$, $(k,l)=1$. We define[1] for $x\geqq 2$

$$P(x,k,l)=\sum_{\substack{p\leq x\\ p\equiv l \pmod k}} \alpha_p,$$

where

$$\alpha_p=\log p\ \exp\left(-p\frac{\log x}{x}\right),$$

*) Computation Centre, Hokkaidô University.

**) Department of Mathematics, Hokkaidô University.

1) Throughout in this note the letters p and q are used to represent prime numbers.