

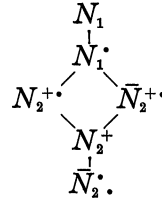
8. On Newman Algebras. II

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3. The Equational Basis **B**. To show the equational completeness of system **B**, it will suffice to derive $\bar{N}_2^+$ from it, because the $^{++}$ -transforms of the equations of **B** and $\bar{N}_2^+$ yield precisely Wooyenaka's axiom system II (see [7] and [8]):



This implies then that $\mathbf{B}^{++}$ is an equational basis for Newman algebras and the superfluousness of $\bar{N}_2^*$ in Wooyenaka's system II.

3.1. $xx = x.$

$$x = x(x + \bar{x}) = xx + x\bar{x} = xx \quad (N_2, N_1, \bar{N}_2).$$

3.2. $x\bar{x} = \bar{x}.$

$$x\bar{x} = x\bar{x} + \bar{x}\bar{x} = (x + \bar{x})\bar{x} = \bar{x} \quad (\bar{N}_2, N_1^*, N_2^*).$$

3.3. $x + \bar{x} = y + \bar{y}.$

$$x + \bar{x} = (x + \bar{x})(y + \bar{y}) = y + \bar{y} \quad (N_2, N_2^*).$$

3.4. $x + \bar{x} = \bar{x} + x.$

(a) $(\bar{x} + x)\bar{x} = \bar{x}\bar{x} + x\bar{x} = \bar{x}\bar{x} + \bar{x} = \bar{x}\bar{x} + \bar{x}\bar{x} = (\bar{x} + \bar{x})\bar{x} = \bar{x} \quad (N_1^*, 3.2, 3.1, N_1^*, N_2^*).$

(b) $(\bar{x} + x)\bar{x} = \bar{x}\bar{x} + x\bar{x} = \bar{x}\bar{x} = \bar{x} \quad (N_1^*, \bar{N}_2, 3.1).$

Then $x + \bar{x} = \bar{x} + \bar{x} = (\bar{x} + x)\bar{x} + (\bar{x} + x)\bar{x} = (\bar{x} + x)(\bar{x} + \bar{x}) = \bar{x} + x \quad (3.3, (a)-(b), N_1, N_2).$

3.5. $\bar{\bar{x}} = x.$

$$\bar{\bar{x}} = x\bar{x} = x\bar{x} + x\bar{x} = x(\bar{x} + \bar{x}) = x(\bar{x} + \bar{x}) = x \quad (3.2, \bar{N}_2, N_1, 3.4, N_2).$$

3.6. $(y\bar{y})(\bar{y}\bar{y}) = y\bar{y}.$

$$\begin{aligned}
 (y\bar{y})(\bar{y}\bar{y}) &= (y\bar{y})(\bar{y}\bar{y}) + y\bar{y} = (y\bar{y})(\bar{y}\bar{y}) + (y\bar{y})^2 = (y\bar{y})(\bar{y}\bar{y} + y\bar{y}) \\
 &= (y\bar{y})(y\bar{y} + \bar{y}\bar{y}) = y\bar{y} \quad (\bar{N}_2, 3.1, N_1, 3.4, N_2).
 \end{aligned}$$

3.7. $\bar{y}\bar{y} = y + \bar{y}.$

$$\bar{y}\bar{y} = (y\bar{y} + \bar{y}\bar{y})(\bar{y}\bar{y}) = (y\bar{y})(\bar{y}\bar{y}) + (\bar{y}\bar{y})^2 = y\bar{y} + \bar{y}\bar{y} = y + \bar{y} \quad (N_2^*, N_1^*, 3.6-3.1, 3.3).$$

3.8. $x\bar{x} = y\bar{y} \quad (3.5, 3.7, 3.3, 3.7, 3.5).$

3.9. $x(y\bar{y}) = y\bar{y}.$

$$\begin{aligned}
 x(y\bar{y}) &= x(x\bar{x}) = x(x\bar{x}) + x\bar{x} = x(x\bar{x} + \bar{x}) = x(x\bar{x} + \bar{x}\bar{x}) \\
 &= x((x + \bar{x})\bar{x}) = x\bar{x} = y\bar{y} \quad (3.8, \bar{N}_2, N_1, 3.1, N_1^*, N_2^*, 3.8)
 \end{aligned}$$